Temperature, Frequency, and Young's Modulus of an Aluminum Tuning Fork

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Abstract

The frequency produced by a standard C (523.3 Hz) aluminum alloy tuning fork when struck at temperatures ranging from 29°C to 300°C was studied. It was found that frequency decreased with increasing temperature with an inverse exponential relationship. The frequency was used to calculate Young's Modulus for aluminum, with the results being in close agreement with published values.

Introduction

The frequency of a tuning fork depends on its dimensions and the density and elasticity of the material of which it is made. The elasticity of metal is what causes tuning forks to resonate. Young's Modulus, a measure of the elasticity of the material, is defined as the ratio of the stress applied to a body and the resulting strain. The equation used to find the frequency of a tuning fork is[1]

$$f = \frac{1}{2\pi l^2}\sqrt{\frac{AE}{\rho}} \qquad \text{(Equation 1)}$$

where f is the frequency of the fork, A is the cross-sectional area of the prongs (tines), l is the length of the prongs, E is the Young's modulus of the material the fork is made from, and ρ is the density of the fork. It should be noted that given the shape of a tuning fork, the effective length of the prongs is somewhere between the length of the inner and outer edges of the prongs.

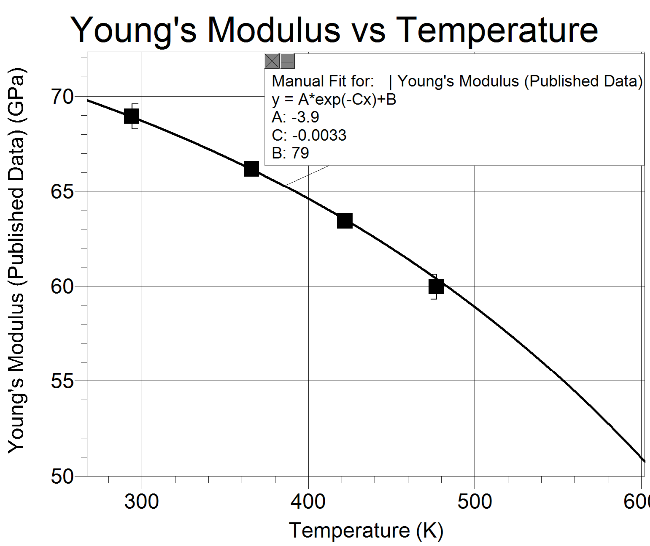

Figure 1 Published data[2] on the dependence of Young's Modulus on temperature for temperatures up to 480 K.

The elasticity of metal depends on its temperature. According to published data[2] the relationship between Young's Modulus and the temperature of aluminum can be represented as

$$E = -3.9\, e^{-0.0033\, T} + 79 \qquad \text{(Equation 2)}$$

where E is Young's Modulus in GigaPascals and T is temperature in Kelvin.

Given equations 1 and 2, it is predicted that frequency will decrease with increasing temperature with an inverse exponential relationship. In addition, using published values for Young's Modulus, the effective length of the prongs will be determined and from that Young's Modulus can be determined for temperatures up to 570 K.

The results of this research will be useful to any piano tuner who is going to work in hell and needs to know the frequency of the tuning forks at the very high temperatures typically found there. It might also be useful to the performer who recently became famous for setting a piano on fire and playing it until the strings inside the piano broke.[3]

Methods

The tuning fork that was used in the experiment was made out of aluminum which resonated at a frequency of 523.3 Hz, or the note C at room temperature. The dimensions of the tuning fork are shown in figure 2.

A thermocouple was secured to the base of the tuning fork and was wrapped with flame resistant cloth. A microphone was clamped near the tuning fork. Before the tuning fork was heated the tuning fork was struck and the sound recorded at 29°C. Three Bunsen burners were placed at equal intervals approximately 15 cm underneath the tuning fork. The tuning fork was gently heated till it reached a temperature above 300˚C. The tuning fork was then taken away from the burners and was struck. The sound was recorded at 100,000 samples per second with a Vernier microphone and the frequency determined using a Fast Fourier Transform. The temperature of the thermocouple at the time of each recording was noted. The tuning fork was struck and the sound recorded periodically as it cooled, noting temperature each time. This was done until the tuning fork reached a temperature of around 60° C. It was then put over the Bunsen burners again and reheated till it reached a temperature above 300 ˚C and the process was repeated. This was done a total of 4 times.

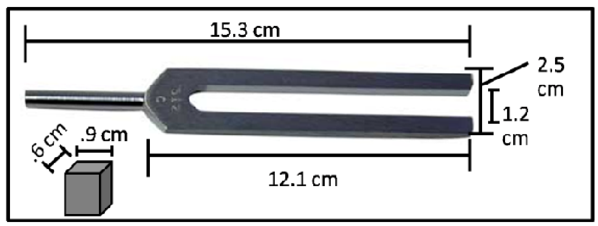

Figure 2 The dimensions of the tuning fork used.

Results and Discussion

As can be seen in figure 3 the relationship between temperature and the frequency is a negative exponential relationship. The relationship is given as

$$f = -\,8.40\; e^{0.00415\,T} + 553 \quad \text{(Equation 3)}$$

where f is frequency and T is the temperature of the tuning fork in Kelvin.

It is interesting to note that there seems to be an increase in the rate of frequency

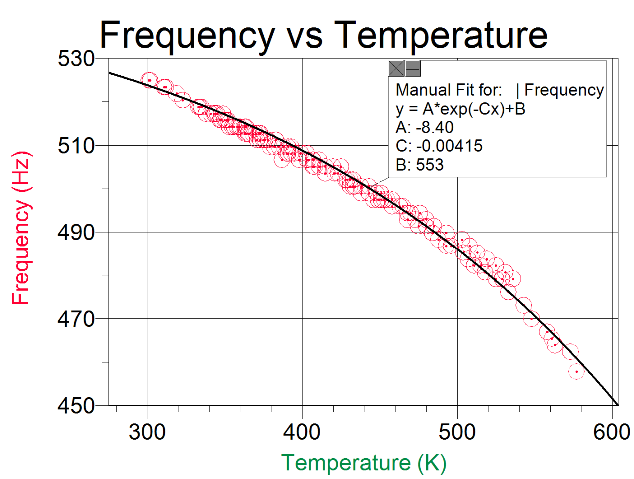

Figure 3 Frequency vs Temperature for all four test runs.

drop above 530 K. This may be due to changes in the crystal structure of aluminum above this temperature. It was noted that the prongs of the fork were slightly warped after the experiment was completed. Another point that can be noted is the consistency of the four temperature runs. Even though there was warping after the first run, the data remained consistent for the rest of the runs. After the final run, the tuning fork was cooled back to room temperature. When the fork was stuck again, the fork resonated at the same frequency as it did before it was heated. This seems to indicate that even when aluminum is heated to high temperatures and warping occurs, Young's modulus returns to the original values after it cools.

In the theoretically derived equation 1, the length of the prongs is defined as from the tip of the prong to where the metal in the prong experiences zero displacement. In a tuning fork, this length is different for the inside and outside of the prongs. To determine the effective length of the prong, the published value of Young's Modulus at 29ºC was used in equation 1 along with the measured dimensions and frequency of the tuning fork at that temperature. The effective length of the prong was determined to be 0.115 m which is a reasonable value based on the dimensions shown in figure 2.

Once the effective prong length was determined, Young's Modulus was calculated for the entire range of temperatures tested, as shown in figure 3. The experimental values are in close agreement with published data within the limited temperature range of the published data, confirming the validity of this method of determining Young's Modulus. Using the frequency of the tuning forks allows us to determine Young's Modulus over an increased temperature range.

Equation 2 is shown to be a valid model for the relationship between temperature and Young's Modulus of aluminum, and has extended the range of validity from 480 K up to 580 K. It will be difficult to use this method to determine Young's Modulus for aluminum above this temperature as it was noted that the amplitude of resonance of the tuning fork dropped off significantly above 580 K.

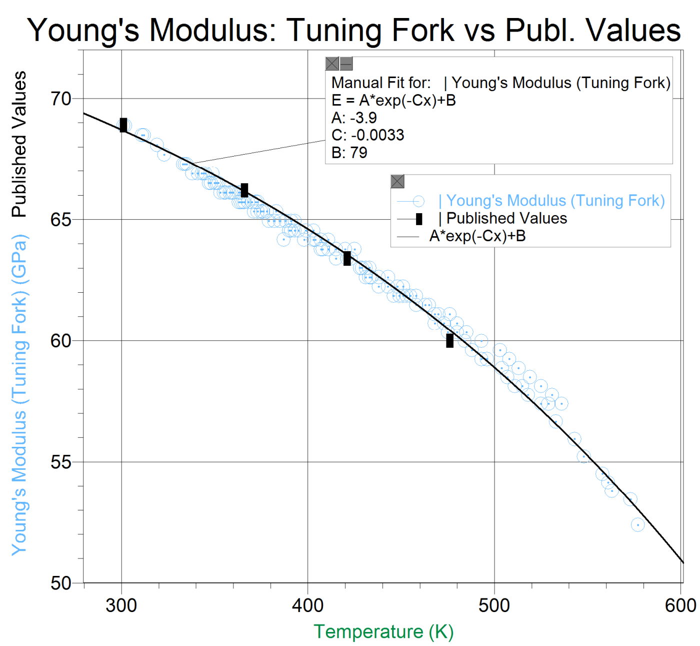

Figure 3 Young's Modulus of aluminum derived from the frequency of the tuning fork. Note the close agreement with published values.

One issue was that as the fork cooled, the tips probably cooled faster than the base where the thermocouple was located, so the temperature recorded by the thermocouple was not the temperature of the entire tuning fork. A method to ensure uniform temperature across the entire tuning fork is needed.

Further research is suggested with tuning forks of various materials. Also, the frequency of a variety of clamped bars at different temperatures could be tested to confirm the validity of this method of determining Young's Modulus.

Conclusion

It has been shown that the temperature affects the frequency of a tuning fork in a negative exponential relationship, as shown in equation 2. Also, when aluminum is heated to nearly 600° Kelvin, Young's Modulus returns to its original value after the aluminum cools again. Finally, it was shown that the frequency of a tuning fork can be used to find Young's Modulus at high temperatures given a known value for Young's Modulus.

References

[1] *Tuning Fork*. Wikipedia. Retrieved December 21, 2011, from http://en.wikipedia.org/wiki/Tuning_fork

[2] *Young's Modulus of Elasticity for Metals and Alloys.* The Engineering Toolbox. Retreived December 15, 2011, from http://www.engineeringtoolbox.com/young-modulus-d_773.html

[3] *Ama 2009 : Lady Gaga Piano On Fire*. King of People. Retrieved January 20, 2011, from http://www.kingofpeople.com/4401/ama-2009-lady-gaga-piano-on-fire/