

Paintball velocity as a function of distance traveled

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Abstract

The relationship between the distance a paintball travels through air and its velocity is investigated by firing a paintball into a ballistic pendulum from a range of distances. The motion of the pendulum was filmed and analyzed by using video analysis software. The velocity of the paintball on impact was calculated from the maximum horizontal displacement of the pendulum. It is shown that the velocity of a paintball decreases exponentially with distance traveled, as expected. The average muzzle velocity of the paint balls is found with an estimate of the drag coefficient.

Introduction

The velocity of a paintball decreases as it travels due to the drag force caused by air resistance. Measurements will be made to determine the relationship between the distance traveled and the velocity on impact using a ballistic pendulum.

Figure 1 shows the forces acting on a paintball after it has been fired. The force due to gravity is neglected because only the horizontal component of the momentum is transferred to the pendulum.

The relationship between the drag force and the velocity of an object traveling through air is

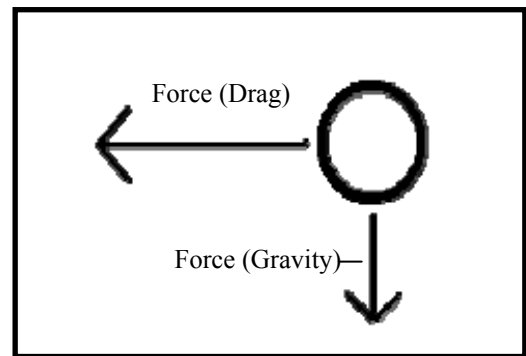


Figure 1 The forces acting on a paintball during flight neglecting buoyancy force.

$$F_D = \frac{C_D \rho v^2 A}{2} \quad (\text{Equation 1})$$

where F_D is the drag force, C_D is the drag coefficient, ρ is the density of air, v is the velocity, and A is the frontal area¹. The drag force is proportional to the square of the velocity.

Equating equation 1 with Newton's second law of motion gives

$$ma = \frac{C_D \rho v^2 A}{2}$$

It can be shown by using the chain rule that

$$\ln v = \frac{-C_D \rho A}{2m} x + c \quad (\text{Equation 2})$$

where m is the mass, a is the acceleration, v is the velocity, x is the distance traveled, and c is an integration constant.

Consequently, the velocity of the paintball is expected to decrease exponentially as it travels through air, with $\ln v$ having a negative linear relationship with x .

When a paint ball is embedded in a ballistic pendulum the horizontal component of momentum determines the initial horizontal velocity of the pendulum and consequently the maximum height of the swing.

The kinetic energy of the pendulum immediately after the collision is converted to potential energy during the swing of the pendulum. At maximum vertical displacement Δh

$$\frac{1}{2}[m_1 + (n)m_2]v_p^2 = [m_1 + (n)m_2]g\Delta h \quad (\text{Equation 3})$$

where m_1 is the mass of the pendulum, m_2 is the mass of a single paintball, n is the total number of paintballs fired into the pendulum, v_p is the initial velocity of the pendulum after the collision, g is the gravitational constant, and Δh is the maximum vertical displacement.

The velocity of the box as a function of height is given by

$$v_p = \sqrt{2g\Delta h} \quad (\text{Equation 4})$$

The horizontal momentum of the paintball is completely transferred to the pendulum during the collision. Applying the law of conservation of momentum gives

$$m_2 v_b = [m_1 + (n)m_2]v_p$$

and $v_b = \left[\frac{m_1 + (n)m_2}{m_2} \right] v_p$ (Equation 5)

where v_b is the velocity of the paintball at impact.

Using equations 4 and 5, the velocity of the paintball at impact can be determined by measuring the maximum vertical displacement of the pendulum.

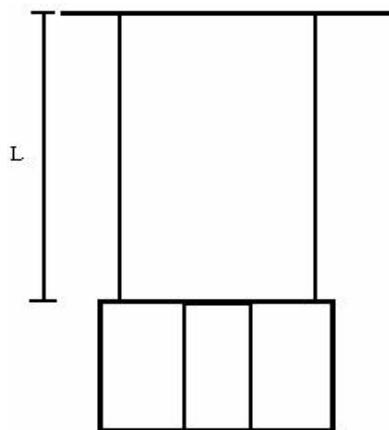


Figure 2 Front-view of the ballistic pendulum. The space in the center is for the paintball to enter.



Figure 3 The paintball gun mounted on a clamp stand.

Methods

A ballistic pendulum was made by cutting a hole in the front of a cardboard box and attaching foam inside the back of the box, to prevent the paintballs from breaking through. Four 253.0 ± 0.5 cm strings were used to attach the box to the ceiling of the room, as shown in figure 2. The strings were attached to the four corners of the box. The box was hung so that the strings were parallel to each other, and perpendicular to the ceiling and the box. The mass of the box was 1100 ± 5 g.

A paintball gun with air pressure of 126 ± 5 psi was clamped to a stand as shown in figure 3, with the barrel parallel to the ground and aligned with the pendulum. Distances with intervals of 50 ± 0.5 cm were measured from the muzzle of the paintball gun to the back of the pendulum where the collision was to take place.

The paintballs were inserted one by one and fired into the pendulum. The number of paintballs fired into the pendulum was counted and added to the mass of the pendulum after each shot. A video camera was placed to the side of the pendulum, perpendicular to the path of the pendulum's swing. The camera recorded the swing of the pendulum after the collision, and the maximum horizontal displacement of the pendulum was measured using Logger Pro video analysis software. The vertical displacement of the pendulum was calculated from the horizontal displacement.

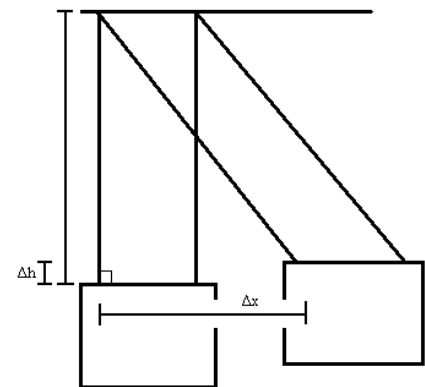


Figure 4 Side view of the pendulum's swing showing the maximum horizontal and vertical displacement of the pendulum.

After each paintball was fired, the pendulum was left to reach equilibrium before the next paintball was fired. Five different distances were tested and four trials were conducted for each distance.

Results and Discussion

Equation 2 gives the natural log of the velocity of the paintball as a function of distance traveled through air as

$$\ln v = \frac{-C_D \rho A}{2m} x + c.$$

From graph 1 the linear equation is

$$\ln v = -(0.06 \pm 0.02)x + (4.45 \pm 0.04). \quad (\text{Equation 6})$$

The y-intercept on figure 5 is 4.45 ± 0.04 . This means that at zero distance (at the muzzle), the natural log of the paintball's velocity is 4.45 ± 0.04 . The muzzle velocity

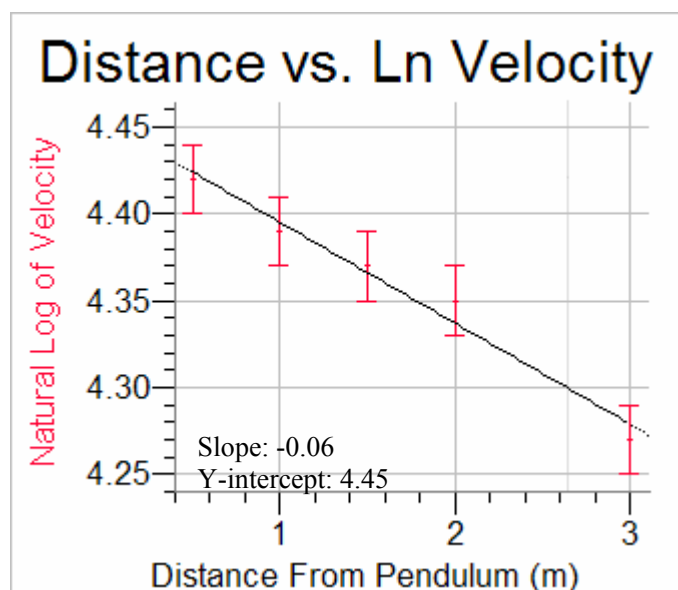


Figure 5 Distance from pendulum versus natural log of the velocity of the paintball with a best linear fit.

of the paintball was therefore 85 ± 3 m/s. The muzzle velocities of typical paintballs are given as 85 to 95 m/s³. The calculated muzzle velocity is at the lower end of this range.

The coefficient of drag was calculated to be 0.3 ± 0.2 by equating the slope of the line in figure 5 with the coefficient of x in Equation 2. C_D for a smooth sphere is given in the literature as 0.5.²

Conclusion and Evaluation

The results showed that the paint ball velocity decreased exponentially as it traveled through the air. The drag force was found to be proportional to the square of the velocity, as predicted by theory. It was also shown that the coefficient of drag of the paintball used was 0.3 ± 0.2 and that the muzzle velocity of the paintball was 85 ± 3 m/s.

There were two major sources of uncertainty in our method. Even though attempts were made to keep the angle of launch and the alignment of the paintball constant, the direction that the paintballs were fired was slightly different for each trial. Some of the paintballs struck the pendulum slightly to the left or right of center. This caused the pendulum to wobble slightly as it swung back in some trials. A method should be devised to better control the firing direction of the gun.

The CO₂ tank was not refilled after each trial. The pressure in the tank would have decreased a small amount. This would have caused the initial velocity of the paintball to decrease with each trial. If the tank were refilled to the same pressure after each trial, the muzzle velocity would be closer to the same for each.

These conclusions are limited to a single type of paintball over distances ranging from 0.5 to 3.0 meters. Measurements could in future be made over a wider range of distances with balls of different mass.

References

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