

Seiche Wave Formation In Deep-Water and Shallow-Water Waves

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Abstract

Seiche waves in a small glass tank partly filled with water were filmed with a digital video camera. The video clips were analyzed with Logger Pro to determine the period of the oscillations at different water depths. The calculated velocity of surface gravity waves in the region of transition between the deep and shallow water approximations was found to agree with theoretical predictions.

Introduction

A seiche wave is a standing wave oscillating in an enclosed body of water¹ (figure 1). Higher harmonics can be generated but only the fundamental was investigated in this research. As figure 1 shows, the fundamental has antinodes at the walls of the tank and a node in the center. The water surface at the center remains at a constant height while the surface at the walls oscillates up and down.

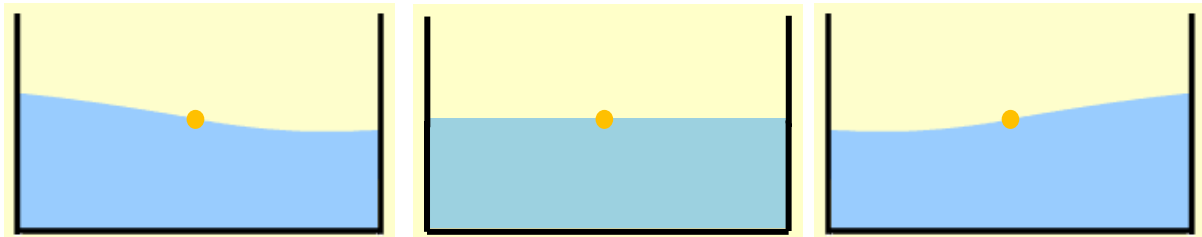


Figure 1 Water displacement at three points during the oscillation³. The orange dot indicates the center of the wave.

By definition, a deep-water wave has a wavelength that is less than half of the water depth. A shallow-water wave has a wavelength that is more than twenty times the water depth². The velocity of deep-water waves is given by

$$v = \sqrt{\frac{g\lambda}{2\pi}} \quad (\text{Equation 1})$$

where v is the velocity, g is the gravitational constant, and λ is the wavelength. As the equation shows, the velocity is independent of the depth of the tank and the seiche wave period is expected to be independent of depth in the deep-water region.

Shallow-water wave velocity is given by

$$v = \sqrt{gh} \quad (\text{Equation 2})$$

where h is the depth of the water⁴. The frequency of a seiche wave is expected to be proportional to the square root of the water depth in shallow water.

Seiche waves for which $\lambda=2L$ were generated by rocking the tank from side to side.

For surface gravity waves

$$v = \frac{2L}{T} \quad (\text{Equation 3})$$

where v is velocity of the wave, L is the length of the water tank, and T is the period³.

Methods

The tank was 0.198 m long and 0.154 m wide. The depth of the water was varied from 0.021 m to 0.082 m. The tank was rocked from side to side to create a seiche wave, as shown in figures 2 and 3. This process was repeated for water depths ranging from the deep-water region to the shallow-water region.

A camera was used to film the movement of the seiche wave. Using the Logger Pro video analysis software, the video clips were analyzed and position-time graphs for one anti-node of each seiche wave were plotted. A damped sine curve was fitted to each position-time graph to determine the period of the oscillation. From this, the velocity of the wave forming the seiche wave was determined and graphed as a function of depth.

Results and Discussion

The oscillations were found to have a constant period independent of amplitude (figure 4). Since the amplitude has no affect on the period, the seiche wave oscillation is simple harmonic.

In deep-water, a seiche wave was formed directly as the tank was rocked back and forth. In shallow-water, a single pulse was first formed that traveled across the tank when the tank was rocked. After several reflections across the tank, the pulse

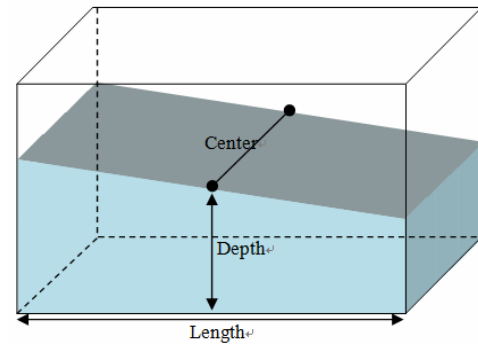


Figure 2 The water tank was rocked back and forth to set up a fundamental seiche wave.

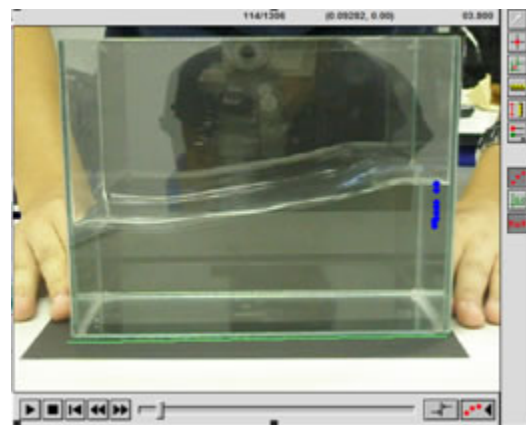


Figure 3 View of the tank when the depth of the water is 0.065 m. The blue dots show the position of the surface of the water at each successive frame.

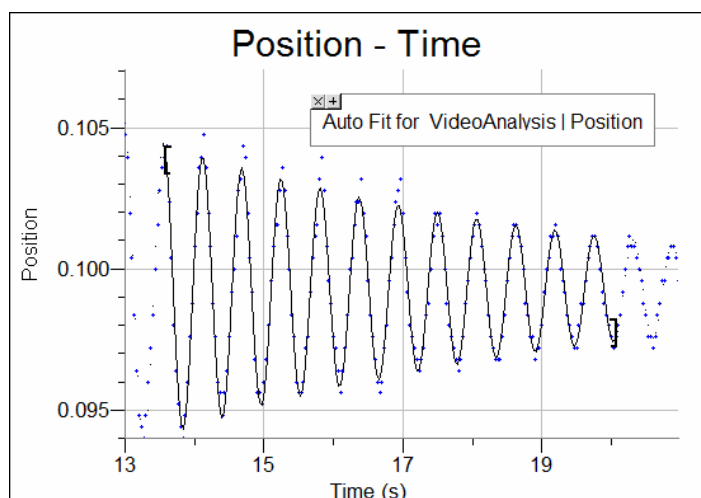


Figure 4 Position-time graph of a seiche wave for a depth of 0.065 m.

transitioned into a low amplitude seiche wave, as shown in figure 5. The maximum amplitude of the seiche wave in shallow water was only a few millimeters.

When water depth is plotted against the calculated velocity of surface waves, an exponential curve can be fitted to the data, as shown in figure 6.

The relationship between velocity v and depth d is

$$v = (-0.71 \text{ m/s})e^{(-33/\text{m})d} + 0.79 \text{ m/s} \quad (\text{Equation 4})$$

While there is no theoretical reason to predict that the relationship between wave velocity and depth in the transition region is an exponential curve, it clearly fits the data well.

Based on Equation 1, the velocity of surface gravity waves in deep-water is expected to be 0.786 m/s when λ is 0.396 m. This velocity is independent of the depth of the water.

The experimentally determined velocity (0.79m/s) is equal to this value within uncertainties.

According to equation 2, the velocity of shallow-water waves is proportional to the square root of the depth. For the tank used, shallow-water seiche waves occur at a depth of less than 0.010 m. The shallowest depth tested was 0.021 m. Equation 2 predicts a theoretical wave velocity of 0.454 m/s for a depth of 0.021m. The experimentally determined wave velocity at 0.021 m is 0.43 m/s. The two values differ by only 4%, showing that 0.02 m is close to the shallow water region.

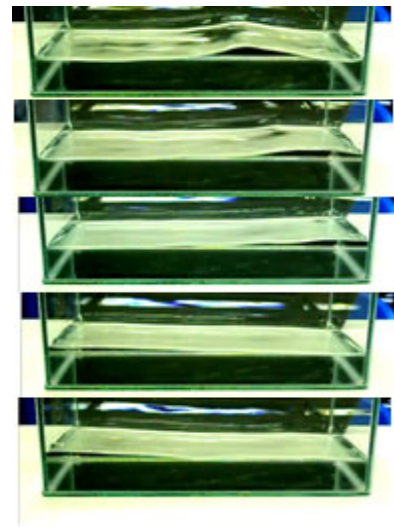


Figure 5 A series of frames from the movie clip when the depth was 0.021m.

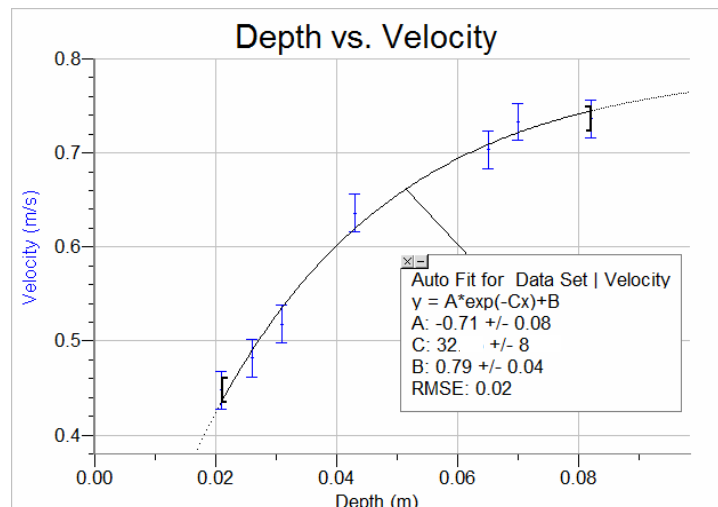


Figure 6 Calculated gravity wave velocity versus depth.

Conclusion and Evaluation

In shallow-water, an initial pulse is formed which transitions to a very low amplitude seiche wave. The maximum amplitude of shallow-water seiche waves was very small but the periods of all oscillations were found to be independent of amplitude (figure 5). Traveling gravity wave velocities, calculated from wavelength and period data, were found to agree with theoretical values based on the deep and shallow water approximations.

In future work a high definition video camera with a macro function would allow a more accurate determination of the period of small amplitude waves in shallow water.

Figure 6 shows only the transition region between deep-water waves and shallow-water waves. Seiche waves fully in the deep-water region and in the shallow-water region could be studied. A longer tank could be used for examining shallow-water seiche waves and the transition from the initial pulse to the seiche wave could be investigated.

References

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