Spin Rate and Gyration Angle of a Cylinder

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Abstract
The relationship between the height of a spinning cylinder and its rotational period and gyration angle was investigated, with the results compared to the predictions of Batista's theoretical model for thin discs.[1] Five 25 mm radius discs with heights ranging from 25 mm to 100 mm were spun on a smooth granite surface and video recorded at 480 frames per second. The Tracker video analysis program was used to determine the period of rotation as a function of the angle of the disc with respect to the surface. It was found that the theoretical model, which assumes negligible disc height, is not applicable when the height exceeds the radius of the discs.

Keywords: spinning cylinder, height to radius ratio, period of precession, angle

I. INTRODUCTION

Euler's Disc is a physics toy that demonstrates concepts such as torque, rotational inertia, and momentum.[2] It consists of a flat, circular disc that spins similarly to a coin. As the disc spins, it undergoes a combined motion of azimuthal rotation and axial precession. Azimuthal angular velocity (ω_s in Figure 1) refers to how fast the disc rotates around its own axis, measured in radians per second. Axial precession angular velocity, (ω_p) which occurs around the y-axis and represents the rate at which the disc's tilt rotates, is also measured in radians per second.[3]

As energy is lost to friction at the contact point and to air resistance, the disc's angle with the surface decreases. This decrease is accompanied by a decrease in azimuthal rotation and a corresponding increase in the rate of axial precession, as shown in Figure 1.

Batista[1] models the steady motion of a thin disc as it rolls on its edge on a horizontal surface under the influence of gravity as:

$$T^2 = \frac{\pi^2 r \sin\theta}{g} \qquad (1)$$

where T is the precession period (in seconds), r is the radius of the disc, g is gravitational acceleration, and θ is the angle between the base of the disc and the surface.

Although Batista's model assumes a disc with negligible height and no friction in its theoretical formula, real spinning discs experience both rolling friction at the contact point and air resistance. Multiple studies have shown, however, that it can

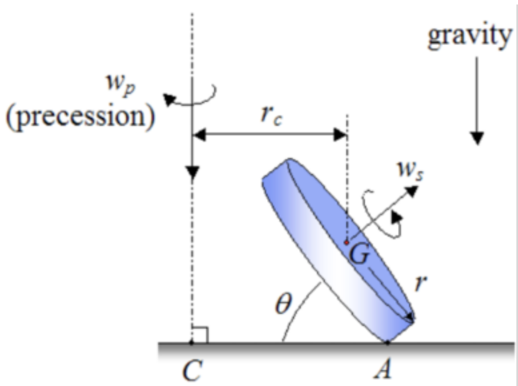

Figure 1. A disc spinning on a flat surface.[4]

approximately describe the behavior of real-world discs. Yun[5] tested the model using a thin glass disc and found only slight deviations from the theoretical predictions, mainly at very extreme angles, likely due to friction. Subsequently, Chulpaibul et al.[6] investigated how disc thickness, or height, influences the validity of Batista's model and concluded that the model remains accurate for discs with height up to the radius, with the proportionality constant between T^2 and $sin\ \theta$ remaining nearly unchanged.

In this study, we extend previous work by analyzing how the proportionality constant between T^2 and $sin\ \theta$ behaves for discs with heights much greater than the radius. The proportionality constant between T^2 and $sin\ \theta$ was calculated for discs with heights ranging from one to four times the radius and compared with the theoretical value of $\frac{\pi^2 r sin\ \theta}{g}$ predicted by Batista's model to evaluate its accuracy across different disc geometries.

II. METHODS

Five 25 mm radius discs with heights ranging from 25mm to 100 mm (Figure 2) were spun manually on a polished granite surface. Video was recorded at 480 frames per second, with the camera positioned horizontally at the level of the granite surface.

The Tracker program was used to analyze the videos of the spinning disc by measuring the angle(θ) of the disc relative to the surface at the beginning of each oscillation, as shown in Figure 3, along with the number of frames it took for the disc to complete one oscillation.

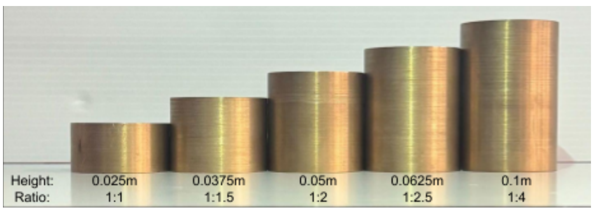

Figure 2. Discs used with height-to-radius ratios ranging from 1.0 to 4.0.

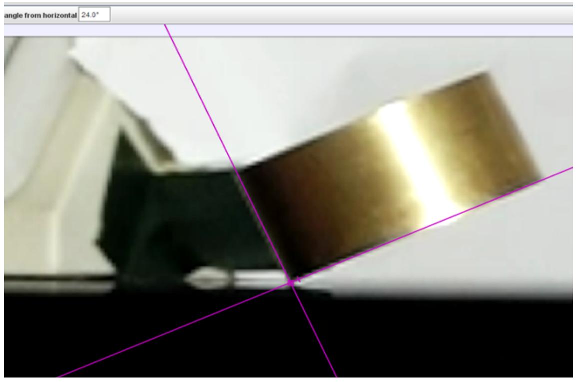

Figure 3. Measuring angle between the spinning disc and the surface.

The proportionality constant between T^2 and $sin\ \theta$ was determined from the data, as shown in Figure 4. For cylinders with greater height, when $sin\ \theta$ is greater than 11.5 degrees, the period of the discs is highly variable as the rotation of the discs became unstable, as seen in Figure 5, thus the period of rotation was only analyzed for angles up to 11.5 degrees.

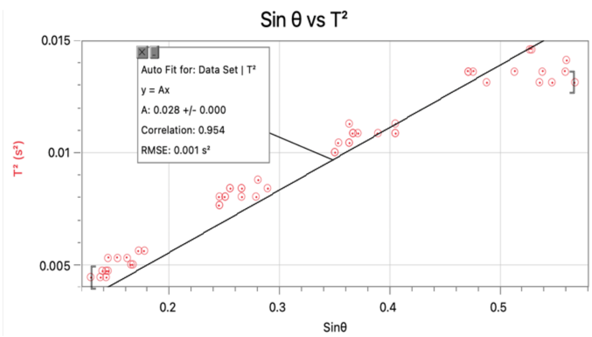

Figure 4. The proportionality constant between T^2 and sin was 0.028 for the first trial of the 25 mm height cylinder.

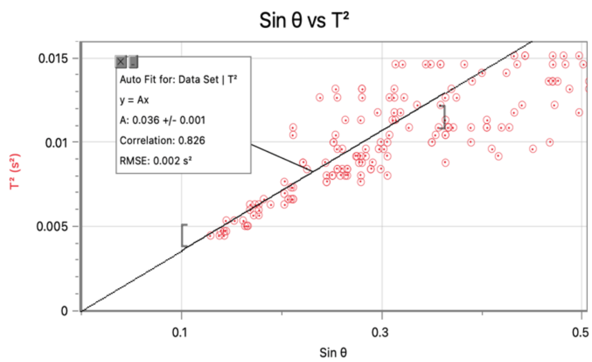

Figure 5. The period of oscillation the discs was unstable for angles with $\boldsymbol{sin\ \theta}$ greater than 0.2

III. RESULTS AND DISCUSSION

Plotting the ratio between cylinder height and radius on the x-axis and $T^2/sin\ \theta$ on the y-axis for its respective ratios can be seen in Figure 6. Batista's theoretical model predicts that $T^2/sin\ \theta$ remains constant for spinning discs, with a proportionality constant of $\frac{\pi^2 r}{g}$. Yun[2] shows that this model is valid for thin discs, as plots of T^2 vs $sin\ \theta$ exhibit a positive linear relationship with the proportionality constant predicted by Batista. Similarly, Chulpaibul et al[3] demonstrated that the model also holds for discs with height-to-radius ratios up to one.

We also show a positive proportional relationship of T^2 vs $sin\ \theta$ for all cylinders tested for angles below 11.5 degrees. This suggests that as the angle of the disc from the surface increases, the period also increases. These results are consistent with Yun's findings, providing further confidence in the validity of the data and its use in exploring the relationship between $T^2/sin\ \theta$ and the disc's height.

The relationship between the height-to-radius ratio and the $T^2/sin\ \theta$ ratio is shown in Figure 6. For the disc with a height-to-radius ratio of 1.0, our results agreed with those reported by Chulpaibun of 0.032. However, for discs with height-to-radius ratios greater than 1.0, the data follows an upward trend, diverging from that predicted by Batista's model and the findings of Chulpaibul's paper. This suggests that for discs with a height-to-radius ratio greater than 1:1, the behavior of the disc does not follow the Batista model.

Our results show that as the height-to-radius ratio increases, the $T^2/sin\ \theta$ increases at an increasing rate. An empirical model, derived from Figure 6, is proposed to describe the behavior of spinning discs with a height-to-radius ratio greater than 1.0:

$$\frac{T^2}{sin\theta} = (0.0057 \pm 0.0007)(\frac{h}{r})^{(2.58 \pm 0.09)} + (0.028 \pm 0.002) \quad (2)$$

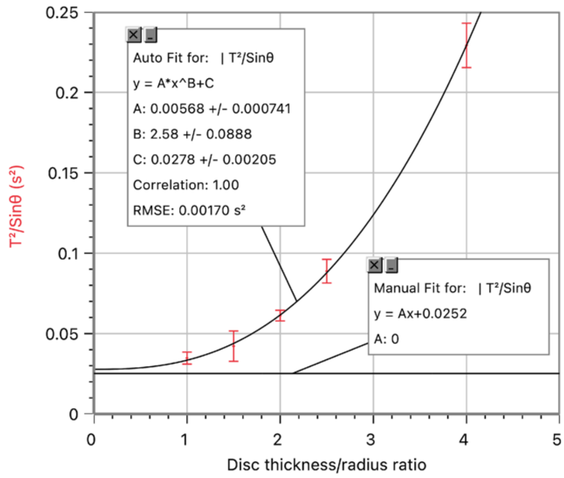

Figure 6 Relationship between $T^2/sin\ \theta$ and the disc height-to-radius ratio.

The empirical model describes the period and angle of discs with heights ranging from 20 to 400% of their radius, while remaining consistent with Batista's theory for thin discs with ratios under one. For ratios approaching zero, $T^2/sin\ \theta$ approaches 0.028, closely matching the theoretical value of 0.025 predicted by Batista's model for thin discs, and the experimental value of 0.026 reported by Yun. For ratios above one, however, $T^2/sin\ \theta$ is dependent on the ratio raised to the power of 2.58, enabling predictions of the behavior of discs with ratios outside of those tested experimentally.

Further research is suggested to develop a theoretical model for discs with height-to-radius ratio greater than one, as this would increase the confidence in the validity of the empirical model proposed here.

IV. CONCLUSION

The effect of increasing the height of a spinning cylinder on the relationship between the period and angle of the disc was tested. For discs with a height-to-radius ratio greater than one, the ratio $T^2/sin\ \theta$ increases at an increasing rate. Hence, it can be concluded that Batista's theoretical model is not applicable for discs where its height exceeds its radius.

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