Water and Frequency of a Tapped Wine Glass

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Abstract

The effect of the height of water in a cylindrical wine glass on the frequency when tapped was investigated. The conditions with water inside, outside, and on both sides of the glass wall, were tested for water heights ranging from no water to nearly the top of the glass. The relative effect of each of the three conditions on the frequency followed that predicted by Chen.[1] The results for each of the three conditions were also shown to follow the model proposed by French[2] for water heights up to 85% full.

Keywords: wineglass, frequency, water level

I. INTRODUCTION

When water is added to a wine glass, the frequency of the sound it makes when tapped decreases. Musicians can produce different notes by varying the water levels in a set of glasses, with some even performing the Sugar Plum Fairy by Tchaikovsky by stroking the rims of an array of glasses in what is described as a glass harp.[1] While research has been done on the change in frequency when the water level varies inside a wine glass, little research exists about the change in frequency when the water is added outside and both inside and outside at the same time.

In 2005, Chen investigated the relationship between wine glasses with various configurations of water added, as illustrated in Figure 1.[2] Chen claims that for a given water depth, the resonance frequency with water added outside of the wine glass will be the highest value, followed by when the water is inside, with the case with water on both sides having the lowest frequency. The frequency shift is defined as the change in frequency from the frequency of the empty glass to when the glass is partially filled with a certain height of water. Chen tested the frequency of a wine glass with water inside, outside, and on both sides at just two different heights. Chen's theory claims that water against the wall will reduce the frequency, compared to the frequency of an empty glass, and his data shows almost the same shift for water on both inside only and outside only.[2]

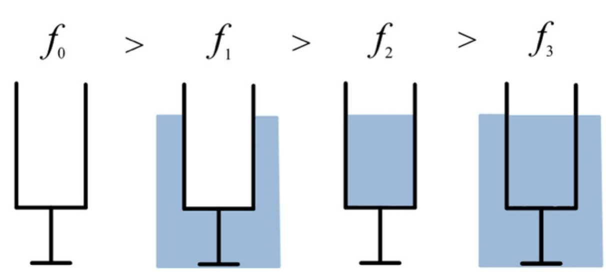

Figure 1. A water-filled wine glass with no water added (*f_0*), water outside (*f_1*), water inside (*f_2*), water on both sides (*f_3*).[2]

However, interestingly, water on both sides simultaneously does not have an additive effect, as the frequency shift for water on both sides is less than that for the sum of inside only and outside only.

Chen's data shows that the ratio between the frequency shift for water on both sides divided by the sum of the frequency shifts for water inside and water outside was smaller when the water level was higher, as shown in Table 1. However, Chen did not report the water levels used for his data or provide information about the wine glass used in the experiment, leaving us with little confidence in the validity of his claims. Here, the trend in this ratio is tested for ten different heights, enabling us to develop a more precise model.

Prior to Chen's measurements, A.P. French proposed a simplified model of the effect of water depth in a cylindrical wine glass on the frequency produced when tapped.[3] His model is presented as:

$$\left(\frac{f_0}{f_d}\right)^2 \approx 1 + \frac{\alpha \rho_l R}{5 \rho_g a}\left(\frac{h}{H}\right)^4 \quad (1)$$

where f_0 is the frequency of the empty glass, f_d is the frequency of the partially filled glass, h is the distance from the bottom of the glass to surface of the water, H is the effective height of the walls of the glass, α is a constant that can be derived from the model, ρ_l is the density of the liquid, ρ_g is the density of the glass, R is the radius of the glass, and a is the glass thickness. As the water height increases, the frequency produced decreases. French showed that equation 1 was a valid model for water inside a cylindrical glass. Figure 2 shows French's result from applying his data to Equation 1.[3] It clearly shows a linear relationship between $\left(\frac{f_0}{f_d}\right)^2$ and $\left(\frac{h}{H}\right)^4$, with a slope of 1.29. Here, the validity of his model for water outside a glass and for water on both sides of a glass will be tested.

	Sum of frequency shifts for water inside only + outside only. (Δf_{i+o}) (Hz)	Frequency shift for water on both sides (Δf_b) (Hz)	Ratio (Δf_b)/ (Δf_{i+o})
Low water level	228	216	0.95
High water level	400	316	0.79

Table 1. Frequency shifts and the ratio for Chen's data for a cylindrical wine glass.

II. METHODS

The water depth filled inside and outside of a cylindrical wine glass was measured using a vernier caliper with an uncertainty of ±0.05 mm. The wine glass had an inner diameter of 43.20 mm, a wall thickness of 2.35 mm near the rim, and an inner wall height of 126.00 mm, as shown in Figure 3. The wine glass was fixed to a retort stand, and a Vernier microphone fixed above the glass vertically, recording with a sample rate of 100,000 samples per second for 0.3 seconds. The sound was recorded as the rim of the wine glass was tapped with a wooden pencil.

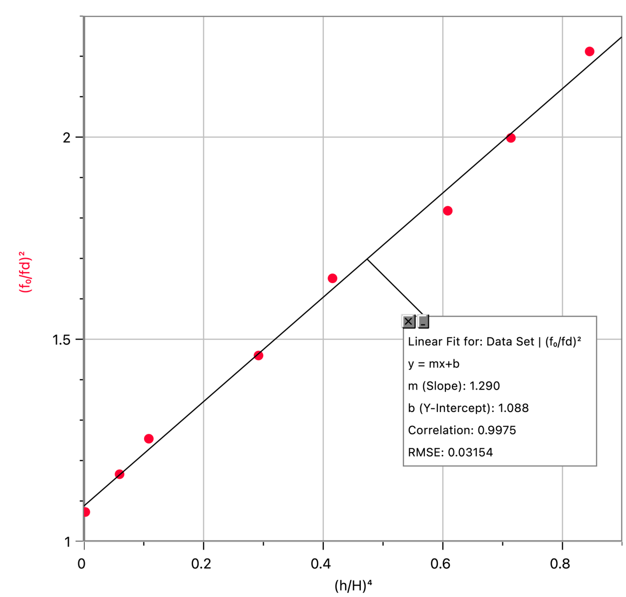

Figure 2. French's result for Equation 1.[3]

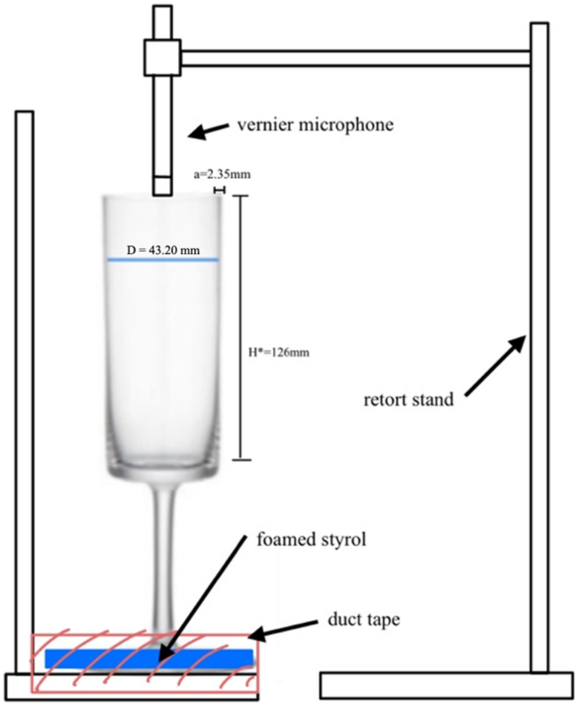

Figure 3. Dimensions of the wine glass and experimental setup.

The frequency of the wine glass was measured for water depths ranging from 0 to 112 mm. The empty glass was then placed in a tank, and the frequency was measured for the same outside depths as were tested for water inside the glass. Finally, the water was added both to the tank and inside the glass, and the frequency was measured with water both inside and outside the glass at the same depths as previously tested.

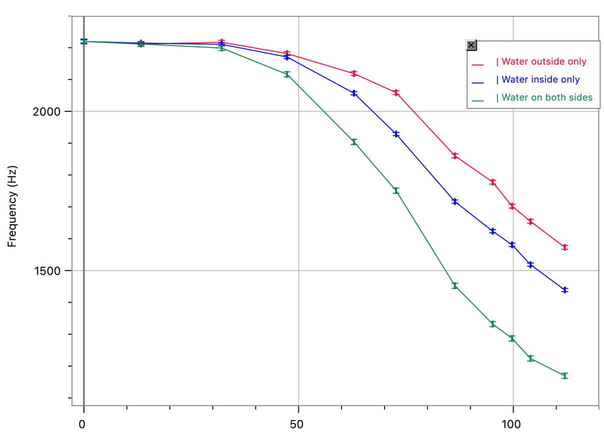

Figure 4. The relationship between the depth of water and the frequency for the three different conditions.

III. RESULTS AND DISCUSSION

The relationship between the height of water filled inside, outside, and on both sides of the wine glass and its frequency when tapped is shown in Figure 4. The data agreed with Chen's prediction shown in Figure 1, where the resonance frequency with water added outside of the wine glass was the highest value, followed by when the water is inside, and water added on both sides having the lowest frequency. The frequency changes were small at lower water depths, and as the water depth increased to greater than 47.3 mm, the frequency value dropped rapidly. However, as the water depth gets closer to the top of the wine glass, around 95 mm, the trend becomes more linearized, and the rate of change in frequency becomes roughly constant. This can be seen in all three conditions.

The ratio between the frequency shift for water on both sides divided by the sum of the frequency shifts for water inside and water outside was calculated, as shown in Table 2. The table presents four selected data points for which the glass is more than half full, as this is where the water starts to have a significant effect. The relationship between the water depth and the ratio $(\Delta f_b)/(\Delta f_{i+o})$ is shown in Figure 5, leading to an empirical model of the relationship between the ratio (R) and the water depth (d), which is only

Water depth (±0.1 mm)	Sum of frequency shifts for water inside only + outside only (Δf_{i+o}) (±6 Hz)	Frequency shift for water on both sides (Δf_b) (±3 Hz)	Ratio $(\Delta f_b)/(\Delta f_{i+o})$ (±0.02)
62.9	264	316	1.20
86.4	862	767	0.89
99.7	1156	933	0.81
112.0	1427	1050	0.74

Table 2. The frequency shifts and calculated ratio for selected data.

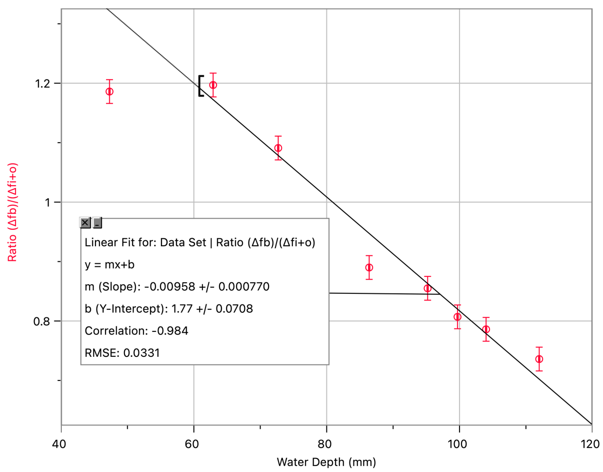

Figure 5. The relationship between the water depth and the ratio (Δf_b)/ (Δf_{i+o}).

It is clear that the gradient of and the α value correlated, as water on both sides has the highest α applicable to the glass more than half full with water:

$$R = (-0.0096 \pm 0.0008)d + (1.77 \pm 0.07) \quad (2)$$

Interestingly, the ratio starts at 1.2 and then slowly drops to 0.7. This shows that the net effect of when the water is added on both sides is changing, where it has more of an effect on frequency than water filled on each side individually for lower water levels, and as the level gets higher, it has progressively less effect than each side individually for depths above around 80 mm, or 2/3 of the glass height. As Chen

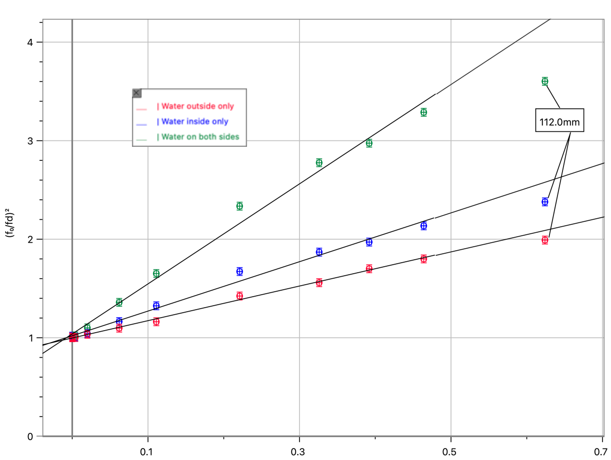

Figure 6. French's linear relationship. Note that the relationship is not valid for the water height above 104 mm.

did not provide enough data, his results and the results here cannot be fully compared. However, it can be noted that both of Chen's results match the value of my experiment above 2/3 of the glass height. Although there is a clear trend for decreasing ratios, no current theory could explain why there is such a trend. Further work is suggested to derive a theoretical model explaining this trend.

French's theoretical model proposes that there is a linear relationship between $(\frac{f_0}{f_d})^2$ and $(\frac{h}{H})^4$. These values were calculated for all depths tested, as shown in Figure 6. The linear fit in Figure 6 shows that French's model is valid for water depths up to 104 mm, or the $(\frac{h}{H})^4$ value of 0.464. However, for all three conditions, the model is not valid at the highest water level, when the water height was 112.0 mm and the $(\frac{h}{H})^4$ value of 0.624, as it does not follow the trendline for the rest of the depths tested.

It should be noted that the wine glass wobbled slightly as it was being tapped, which could have caused the water to oscillate up and down on the walls of the glass. This would result in a small variation in frequency. However, it is unlikely that these variations explain why the model is not valid at higher water levels.

Although all three conditions follow the linear relationship predicted by French, the slope of the graph for water added both inside and outside is the largest, followed by when water is inside only, with the slope for water outside only being the least. Since all the densities and other constants are the same for all conditions, the only value that can be different is the α value in Equation 1. The α value for each condition was calculated from the estimated value of these constants, as shown in Table 3.

Water inside	Water outside	Water on both sides
2.05	1.44	4.15

Table 3. The α values for all three conditions calculated.

The water on both sides has the highest α value, followed by water on the inside only and water on the outside only. The α value indicates the extent of the effect of changing the height of the water on the frequency value. The higher the α value, the more the frequency is affected by the change in the level of water.

IV. CONCLUSION

The effect of the water height for water added inside, outside, and on both sides of a cylindrical wine glass was tested and compared to theoretical models developed by Chen[2] and French.[3] The ratio for water on both sides divided by the sum of the frequency shifts for water inside and water outside was also investigated, clearly showing a trend that decreases as the height of the water increases. It was also shown that the French model was valid for depths up to 85% of the glass height but was not valid for a depth of 112 mm, or 93% of the glass height.

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