Separating Interleaved Notebooks

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Abstract

It is well known that bound books with interleaved pages can be difficult to separate because of frictional forces. In this work a theoretical model is developed for a special case: the force required to separate horizontally oriented *spiral bound notebooks*. It is found that the relationship between the mimimum separation force and the number of interleaved pages closely follows the theoretical model.

Keywords: Interleaved books, static friction, contact surface

I. INTRODUCTION

A popular MythBusters video featured two interleaved phone books which could only be pulled apart using two tanks.[1] The force holding the books together was friction which is caused when two surfaces are pushed against each other. This could not have been caused by only the weight of the pages pushing on the pages below, as the same effect was observed when the books were turned and hung vertically. Alarcon showed that there is a self-generating normal force when bound books are interleaved which presses the pages together creating a "the harder you pull the harder it gets" effect.[2] As shown in figure 1, when pages of a bound book are interleaved, the outer pages are pulled at an inward angle when the books are pulled apart, leading to a perpendicular force pressing the outer pages against the inner pages. This effect will increase with an increasing number of pages, leading to a large increase in the force needed to pull apart interleaved books with many pages. This angle, shown in figure 1, is not present in figure 2, for a spiral bound notebook where the pages are loosely held by the spiral binding. The effect of the inward pulling angle when bound books are interleaved is not expected to be a significant factor for spiral bound books. The important effect is expected to be friction caused by the weight of the pages pushing down on those below. A theoretical model is developed here based on this assumption.

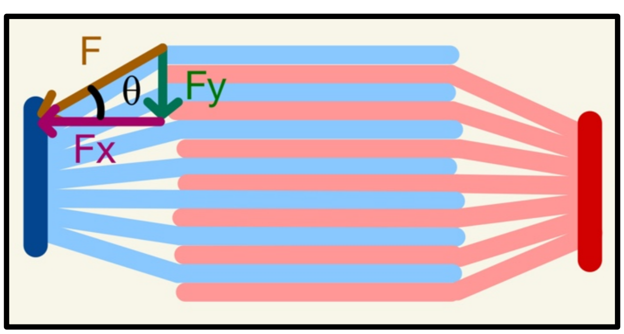

Figure 1. The inward component of the pulling force (Fy) exerted on the outer pages of two interleaved bound books causes increased friction between the pages.

Static friction is the force resisting two unmoving surfaces from sliding over one another.[3] The coefficient of friction (μ_s) relates static friction (F_s) to normal force (F_N), according to the equation,

$$F_s \le \mu_s F_N \tag{1}$$

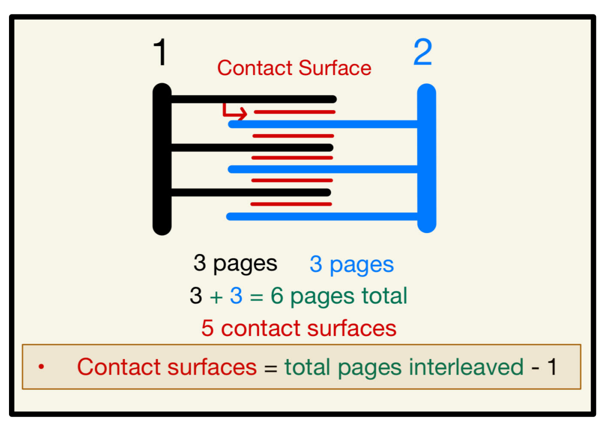

Figure 2. The two notebooks shown are spiral bound, hence the spaced pages.

Applying this concept, interleaved pages of two spiral-bound notebooks experience a static frictional force as the pages are in contact with each other. Because the bottom page of one of the notebooks only comes into contact with the page above it and the table, the number of contact surfaces (M) between pages is always 1 less than the number of interleaved pages (N), as illustrated in figure 2. The normal force (F_N) of the second to top page on the page below it is

$$F_N = m_p g \tag{2}$$

as there is only the mass of one page (m_p) above it. Thus, the frictional force for this page would be

$$F_f = \mu \, m_p \, g \tag{3}$$

As the page below that has the mass of two pages on it, the frictional force would be

$$F_f = \mu(2m_p)g \tag{4}$$

The page below this would have the mass of three pages, and so on. Thus, the total friction force of M+1 interleaved pages would be

$$F_f = \mu(1 + 2 + 3 + \ldots M)m_p g \tag{5}$$

This is equivalent to[4]

$$F_f = \mu\left(\frac{N^2 - N}{2}\right)m_p g \tag{6}$$

The aim is to test whether this model or Alarcon's model best describes the behavior of interleaved spiral bound notebooks.

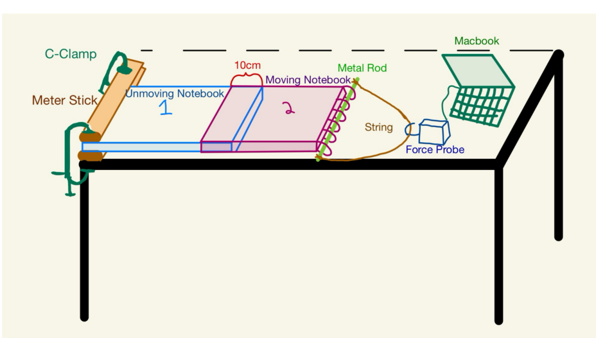

Figure 3. The experimental setup.

II. METHODS

To test if the assumption that Alarcon's model for bound books is applicable to spiral bound notebooks, two notebooks were interleaved and then hung vertically to see if they would slip apart. Instead of pulling them apart, the higher notebook was held with the second suspended from it, free to slip.

To test the validity of the model developed here, the covers of two 22.0 by 26.5 cm spiral bound notebooks were removed, along with unneeded pages. The first notebook was clamped to a table by the spine as shown in figure 3. The second notebook was placed so that it overlapped the unmoving one by 10.0 cm, and 50 pages of each notebook were interleaved. The interleaved part of the page had a mass of 1.52 g.

The moving notebook, notebook 2, had a metal rod threaded through the spine and a string tied to both ends of the rod. A Vernier force probe, recording at 40 samples per second, was pulled gently perpendicular to the spine until notebook 2 started to slip, and the maximum force before slipping recorded. This process was repeated for four more trials with the number of interleaved pages ranging from 50 to 10.

For each trial with a reduced number of pages, the extra pages were removed. All trials were completed in a single session to ensure the humidity of the environment remained relatively constant. The temperature during testing was 27 ± 1°C.

I.

III. RESULTS AND DISCUSSION

It was found that for each notebook with less than 40 pages interleaved, the inward angle effect was so small that the book could not support the weight of the other pages. These results indicate that for spiral bound notebooks, the inward angle effect proposed by Alarcon for bound books is not a major factor, indicating that the model proposed here for spiral bound notebooks, assuming only page weight is likely to be valid to a first approximation.

Regarding the horizontal tests, as the number of pages increased, the force required to separate the notebooks increased at an increasing rate, as shown in figure 4. The relationship can be accurately modeled with the quadratic equation

$$F_A = 0.0054\ (N^2 - N) \qquad (7)$$

This supports the claim that the inward angle effect has a negligible effect on the force needed to pull spiral bound notebooks apart, and the behavior of interleaved spiral bound notebooks can be described by the model developed here. While the positive parabolic relationship fits well for the higher numbers of interleaved pages, the force required for fewer pages trends slightly above the fit. It is possible that there is another factor that has a small impact on the force that has not been accounted for by the proposed model.

Using the A value (0.0054 N) and equation 6, the coefficient of friction of the page was found to be 0.73 ± 0.02. This is higher than a published value of paper-paper friction for notebook paper of 0.55 to 0.59.[5] It is possible that the discrepancy could be due to differences in the type of 'notebook' paper tested, or due to differences in humidity during testing. It is also possible that this is because the meter sticks clamping notebook 1 pressed the pages together, like a bound book, slightly increasing the normal force between the outer pages. It is suggested that force measurements be done for interleaved pages threaded onto a row of straight vertical metal bars, which would eliminate any of the inward force suggested by Alarcon's model. It is also suggested that further research be conducted to determine how the distance that the pages are interleaved affects the force needed to pull the books apart.

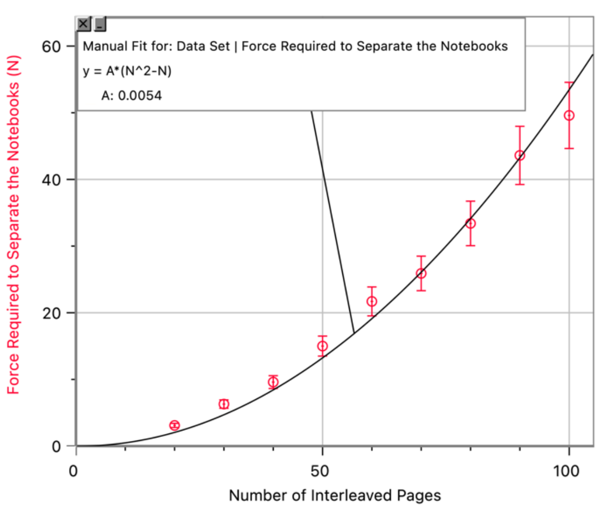

Figure 4. The relationship between average force required to separate the notebooks and number of contact pages.

II. IV. CONCLUSION

The force required to separate interleaved spiral bound notebooks was shown to have a quadratic relationship with the number of pages interleaved. This supported the model proposed here based on only the weight of the pages, not the model proposed by Alarcon for bound books. The coefficient of friction of 0.73 calculated was higher than published values.

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