

Water Layer Thickness and Frequency of a Tapped Wine Glass

C. Laosangfa,¹ A. Raina,² P. Thaipat,² N. Ung-udonpakdee¹

1. Kamnoetvidya Science Academy, 999 Moo 1 Payupnai, Wangchan, Rayong Thailand

2. International School Bangkok, 39/7 Samakee Road, Pakkret, Nonthaburi, Thailand

Email: nirinthanaungudonpakdee@gmail.com

Abstract

Tapping the rim of a wine glass produces a tone. When water is added to the glass, the pitch decreases. The decrease in pitch is attributed to the effective addition of the mass of the liquid to the vibrating walls of the glass. A solid cylinder fixed in the center of the glass reduces the thickness of the layer of water. Chen derived a theoretical model of the effect of the thickness of the water layer touching the walls on the frequency produced, and tested its validity down to a minimum water thickness of 2 cm.⁴ Here, we test Chen's model for water layer thicknesses below 2 cm. Cylinders of different diameters were inserted into the center of a cylindrical glass, while controlling the depth of the water. The sound of the tapped wine glass was recorded and the frequency determined. It was found that Chen's model accurately predicts the frequency for water layer thicknesses above 2 cm, but increasingly diverges for thinner layers.

Keywords: wine glass, frequency, cylinder, radius, water layer

I. INTRODUCTION

The frequency produced by a wine glass when it is tapped decreases as water is added to the glass. Musicians create sounds with different notes by controlling the amount of water in the wine glass, some even play Beethoven's 5th tapping on wine glasses.¹ As this method of creating music has become more popular, scientists have become more interested in the factors that affect the frequency produced.

Studies have been done exploring the effect of increasing amounts of water on frequency,^{2,3} but very few have empirically investigated how the thickness of the layer of water touching the wall of the glass affects the frequency produced.

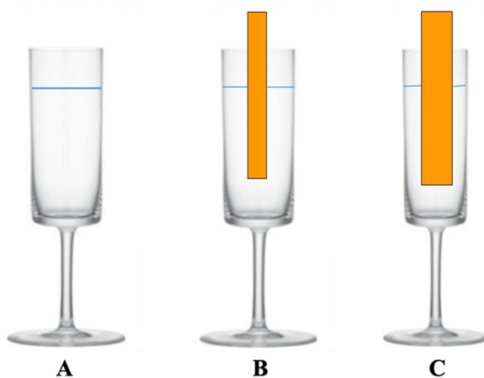


Figure 1. A water-filled cylindrical wine glass with A: no cylinder, B: medium cylinder, C: large cylinder.

When a solid cylinder is inserted and clamped in the center of a cylindrical wine glass partially filled with water, the thickness of the layer of water in contact with the glass walls is reduced, as shown in figure 1.

In 2005, Yih Yuh Chen of the National Taiwan University proposed a simplified theoretical model derived from Hooke's law and Euler's equation.⁴ The relationship between the frequency produced as a function of the radius of the cylinder relative to the radius of a cylindrical wineglass, according to Chen's model, is:

$$\left(\frac{\omega_0^2}{\omega^2} - 1\right) \propto \frac{1+r_1/r_2^4}{1-(r_1/r_2)^4} \quad (1)$$

where ω_0 is the natural frequency of a wine glass with no cylinder inserted, ω is the frequency with a cylinder inserted, r_1 is the radius of the cylinder, and r_2 is the inner radius of the glass.⁴ As the thickness of the layer of water touching the walls decreases, the frequency produced decreases. Chen's model suggests that this phenomenon is caused by the increased fluid pressure between the glass and cylinder, which retards the vibration of the glass, resulting in a lower frequency as the water's thickness decreases. From figure 2, Chen's data shows that for cylinder radii up to 2.0 cm in a glass of radius

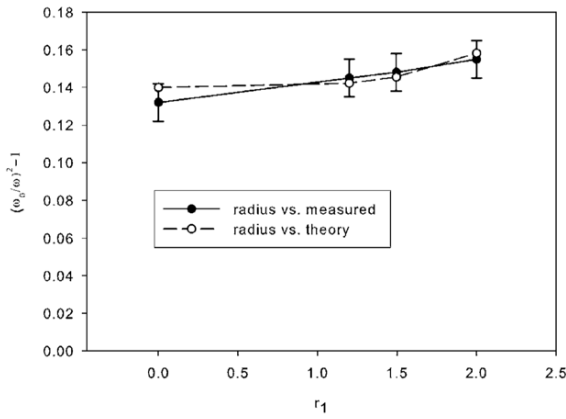


Figure 2. A graph of $(\omega_0/\omega)^2 - 1$ vs. cylinder radius in cm showing values predicted by Chen's model and his experimental values.⁴

4.02 cm, the measured value of $(\omega_0/\omega)^2 - 1$ is equal, within the uncertainty, to the theoretical value from his derived model. Chen only tests cylinders up to one-half the radius (4.02 cm) of the glass (water layer thicknesses down to 2.0 cm), so it is unclear if the model applies to thinner water layers. Here we test the validity of Chen's model for layers of water less than 2.0 cm.

II. METHODS

The thickness of the layer of water touching the inner wall of a wine glass was varied by clamping cylinders of varying diameters in the center of a cylindrical wine glass with an inner diameter of 42.3 mm and an inner wall height of 129 mm. The wineglass secured to a table with the cylinder clamped in the center of the glass, with the bottom of the cylinder was 15 mm above the bottom of the glass. Water was added to the glass until it was 90 mm from the bottom of the glass, as shown in figure 3. A vernier microphone was set up, recording with a sample rate of 100,000 samples per second for 0.3 seconds. The sound was recorded as the rim of the wineglass was continuously tapped with a wooden pencil. The frequency with no cylinder was also measured.

To determine if cylinder material affects frequency, two cylinders made of acrylic and aluminum, with diameters of 11, 24, and 32 mm, were tested, with five trials recorded for each condition. Next, a set of 7 aluminum cylinders ranging in diameter from 11 to 41 mm was tested in the same way. Figure 4 shows the frequencies of the first three resonant modes of the glass.

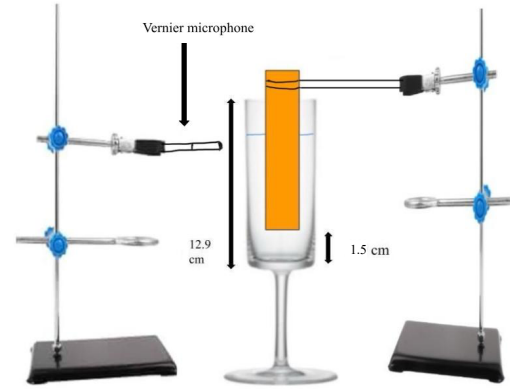


Figure 3. Experimental setup showing the cylinder clamped in the center of the wine glass.

III. RESULTS AND DISCUSSION

There was not a significant difference in the frequency produced using the aluminum and acrylic cylinders, indicating that it is the thickness of the water layer that determines the frequency rather than the nature of the solid cylinder. Thus, testing of the full range of water layer thickness was done using only aluminum cylinders.

The relationship between the average frequency of the first two resonant modes and the diameter of the cylinder can be modeled with an inverse exponential function, as shown in figure 5. For smaller cylinders below 30 mm diameter, there is little effect on the frequency with increasing cylinder diameter. However, as the diameter increases to greater than 30 mm, meaning the diameter of the cylinder is greater than 75% of the glass, the effect of the thickness of the water layer on resonant frequency is increased. As the layer of water gets thinner, below 10 mm, or less than 25% of the glass diameter, the frequency decreases at an increasing rate. This trend was observed in the frequencies of the first 4 resonant modes.

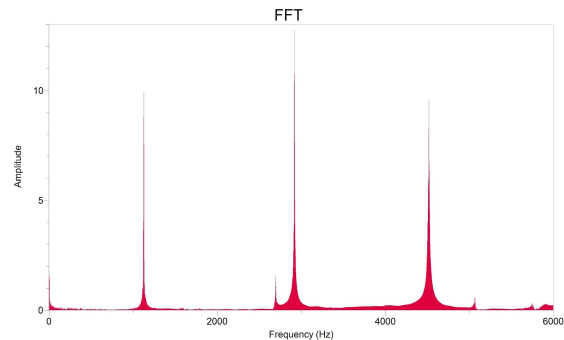


Figure 4. FFT graph of wine glass with 36 mm diameter aluminum cylinder.

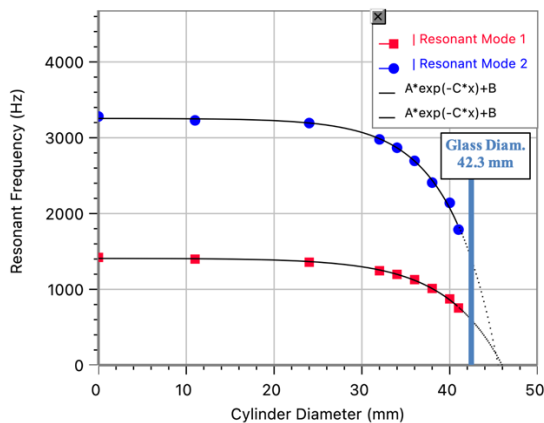


Figure 5. Relationship between the average frequency of the first and second modes of resonance and the diameter of the cylinder.

Chen's theoretical model proposes that there is a proportional relationship between $(\frac{\omega_0^2}{\omega^2} - 1)$ and $\frac{1+r_1/r_2^4}{1-(r_1/r_2)^4}$. The proportional fit in figure 6 shows that Chen's model is valid for cylinders up to 36 mm in diameter, with water layer thicknesses down to 32 mm. Both Chen's results in figure 1 and the results in figure 6 show that his model is valid for water layer thickness greater than 20 mm. However, as the water thickness decreases below 20 mm, the trend increasingly deviates from Chen's theory, as seen in the results for 7 mm and 12 mm thick water layers. It should be noted that the inner diameter of the glass was not uniform and that the cylinder may not have been perfectly centered. This would lead to small variations in the thickness of the water layer, which would become more significant for thinner water layers, however it is unlikely that these variations explain the deviation from the model for the thinner layers.

Chen's model ignores the effect of viscosity, cohesion between water molecules, and adhesion between the water and glass molecules. For very thin layers of water, these factors likely become significant in shaping the behavior of the system. Consequently, a more comprehensive theoretical model that accounts for these factors must be developed in order to explain the results of this paper. Further research is suggested to experimentally determine the effect of fluid viscosity, cohesion, and adhesion on the frequency for water layer thicknesses below 20 mm.

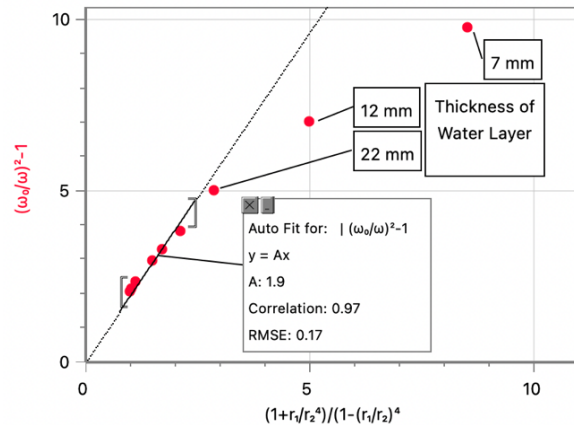


Figure 6. Chen's proportionality relationship. Note that the relationship deviates increasingly from the prediction for water layers less than 20 mm.

IV. CONCLUSION

The effect of the thickness of water in a wine glass on the average peak frequency was tested and compared to Chen's model. The data collected suggests that Chen's theoretical model is only valid for water layers of thicknesses greater than 20 mm. For thicknesses below 20 mm, the frequency deviates increasingly from the frequency predicted by Chen's model.

V. REFERENCES

1. Tiso, R. (2009, April 22). *Glass Harp music-symphony no.5-l.v.beethoven*. YouTube. www.youtube.com/watch?v=gzf9hITzuOM
2. Jundt, G., Radu, A., Fort, E., Duda, J., Vach, H., & Fletcher, N. (2006). Vibrational modes of partly filled wine glasses. *The Journal of the Acoustical Society of America*, 119(6), 3793–3798. doi.org/10.1121/1.2198183
3. Chen, Y.-Y. (2005). Why does water change the pitch of a singing wineglass the way it does? *American Journal of Physics*, 73(11), 1045–1049. doi.org/10.1119/1.2063035
4. Chen, K.W., Wang, C.K., Lu, C.L., & Chen, Y.Y. (2005). Variations on a theme by a singing wineglass. *Europhys. Letters (EPL)*, 70(3), 334–340. doi.org/10.1209/epl/i2004-10493-9