Period and Thickness of a Spinning Disc

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Abstract
The relationship between the thickness of a spinning Euler's Disc and its rotational period was investigated. The study tested Batista's original theoretical model and aimed to include the effect of thickness on the relationship between time period and gyration angle. Five 50 mm diameter discs with thicknesses ranging from 5 to 25 mm were spun on a smooth granite surface, and recorded at 480 frames per second. The Tracker video analysis program was used to determine the average gyration angle and time period of rotation as a function of angle for each disc. It was found that the thickness of the disc does not have a significant impact on the period of the spinning disc for thicknesses up to the radius of the disc.

Keywords: Euler's Disc, spinning disc, disc thickness, spin period

I. INTRODUCTION

Euler's Disc is a physics toy that illustrates principles of torque, rotational inertia, and momentum. It is a flat, circular disc that spins like a coin, as shown in figure 1. As the spinning disc loses energy, the angle of the disc with the surface decreases, which increases its moment of inertia. This has been shown to lead to an increase in the period of rotation, allowing for angular momentum to be conserved.[1,2,3]

The spinning behavior of Euler's Disc is modeled in figure 2, where r represents the radius of the disc, θ represents the angle between the horizontal and the disc, ω_p is the rate of precession in radians per second and ω_s is the rate of spin of the disc in radians per second.

Batista models the steady motion of a rigid disc of negligible thickness rolling on its edge on a horizontal plane under the influence of gravity.[4] The relationship between time period and gyration angle is shown to be,

$$T^2 = \frac{\Pi^2 r sin\theta}{g} \tag{1}$$

where T is the spin period, r is the radius of the disc and θ is the gyration angle. As seen in figure 2 the center of mass of a thick disc is well above the plane where contact with the surface is made. This means that the motion of the center of mass for thick discs with respect to the point of contact will be different than for thin discs. This might have an effect on the validity of Batista's model. To determine the validity of the model

Figure 1. A spinning Euler's Disc

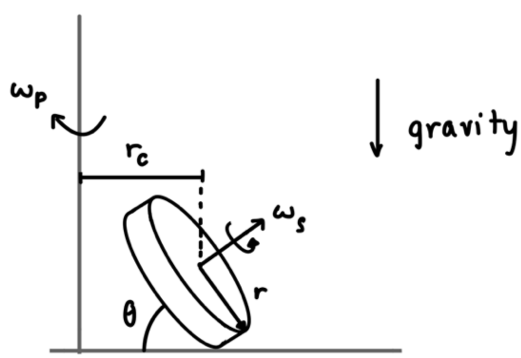

Figure 2. Mechanism of a thick Euler's Disc.

Figure 3. The discs used in the experiment.

we determined the proportionality constant between T^2 and $\sin\theta$ for discs with thickness ranging from 20% to 100% of the disc radius, and compared this to Batista's model.

A study by Zizhou showed that Batista's theoretical model gives a reasonably close approximation to a spinning thin glass disc, however the period of the disc deviated slightly from theoretical predictions, likely due to friction.[5] Here, we aim to build upon Batista's theoretical model with consideration of the thickness of Euler's Disc.

II. METHODS

Five 50-mm-diameter brass discs, with thicknesses ranging from 5 to 25 mm were machined, as shown in figure 3. The discs were spun on a polished granite surface. Video of the spinning discs was recorded at 480 frames per second, with the camera positioned horizontally a few centimeters above the surface.

As shown in figure 4, the protractor tool of the Tracker video analysis program[6] was used to measure the angle between the disc and the surface (θ) at the beginning and end of a spin period to determine the mean angle

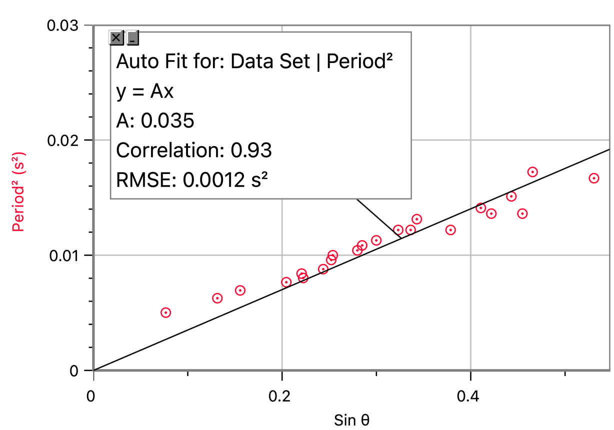

Figure 5. Graph showing how the proportionality constant between T^2 and $\sin\theta$ of one of the trials of the 15 mm thick disc was determined.

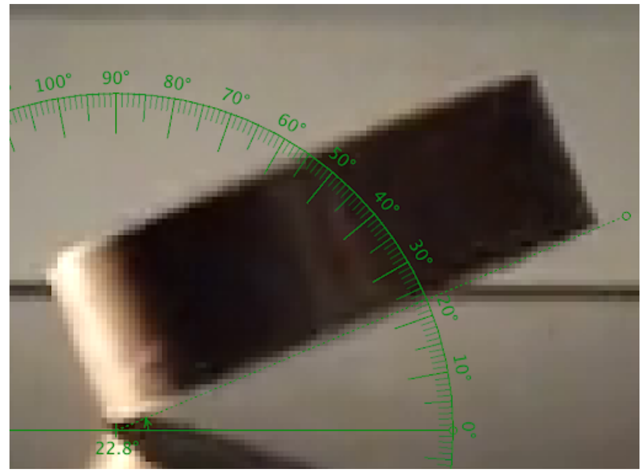

Figure 4. Measuring the angle of the spinning disc.

of the disc at that time. The spin period (T) was determined from the beginning and ending frame of one complete rotation. These measurements were made at intervals as each disc spun until it stopped. An example of the graph used to determine the proportionality constant between T^2 and $\sin\theta$ for the first trial of the 15 mm thick disk is shown in figure 5.

III. RESULTS AND DISCUSSION

The proportionality constant between T^2 and $\sin\theta$ is shown as a function of the ratio of the thickness and radius of the discs in figure 6. As the disc's thickness increases, the proportionality constant between $\sin\theta$ and T^2 stays constant at approximately 0.032 ± 0.001.

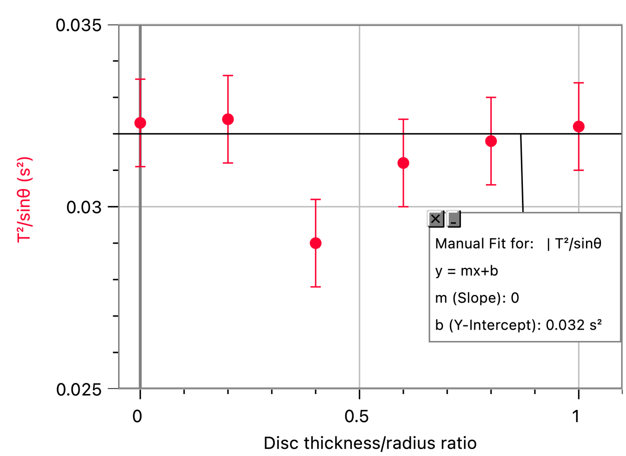

Figure 6. Relationship between the disc thickness/radius ratio and the proportionality constant between $\sin\theta$ and T^2. The 0 value is the theoretical value from Batista's model.

The theoretical model suggests a proportional relationship between $\sin\theta$ and T^2 with a proportionality constant of $\frac{\pi^2 r}{g}$. For the discs used here, this constant is calculated to be 0.0323 ± 0.0001, and is included in figure 6 as the value at a thickness/radius value of zero. The value of the proportionality constant between T^2 and $sin\,\theta$ for the 15 mm disc is much lower than the other discs tested. The reasons for this are not known. It is likely that this is due to differences in the physical characteristics of the disc, however no differences could be seen when the discs were compared.

Further research is suggested for discs with thicknesses much greater than the radius, more like ‘cans’ than ‘discs’, to determine if Batista’s model holds for very thick discs. It is expected that the model will not hold for very thick discs as the motion of the center of mass with respect to the base of a ‘can’ is very different compared to that of a thin disc.

IV. CONCLUSION

The effect of the thickness of a spinning disc on the relationship between the period and angle of spin was tested. It was shown that Batista’s theoretical model for discs of negligible thickness is valid for spinning discs with thicknesses up to the radius of the disc.

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