Temperature and Frequency of a Nyltech Ukulele String

Minyada Chotichaicharin

International School Bangkok,
39/7 Samakee Rd, Pakkret,
Nonthaburi, 11120, Thailand
Email: mminniech@gmail.com

Abstract

A wooden ukulele with nyltech strings was plucked at temperatures ranging from -10°C to 67°C and the frequency was recorded. The thermal expansion and the Young's Modulus of the ukulele string was measured over the same range of temperatures. A theoretical model of the frequency of oscillation of a stretched string as a function of thermal expansion and Young's Modulus was developed. The frequency predicted by the model was compared to the recorded frequency values, supporting the validity of the theoretical model.

I. INTRODUCTION

A ukulele, commonly strung with nyltech, is an instrument that is played around the world. Before playing, a ukulele, shown in figure 1, needs to be tuned, mainly because changes in weather conditions cause a change in the tension of the strings. The most important factor that affects the frequency of nylon strings is temperature change.[1] So, how does temperature affect the frequency produced by the nyltech strings of a ukulele?

Nyltech is a proprietary combination of three synthetic materials, one of which is nylon. Nyltech strings are said to 'settle' more quickly than nylon strings and to hold a tune longer than nylon strings, due to different tension and stretching properties.[2] "Concert Ukulele Nyltech" strings, manufactured by the D'Addario company, were used for this investigation.

Figure 1. Ukulele used in this investigation

The frequency produced by the first harmonic (Figure 2) of a vibrating ukulele string is given by:

$$f = \frac{v}{2L} \tag{1}$$

where f is the frequency produced, v is the velocity of the wave traveling along the string, and L is the length of the stretched string.

The velocity of a wave traveling along a stretched string is given by:

$$v = \sqrt{\frac{T}{m/L}} \tag{2}$$

where m is the mass of the string, and T is the tension of the taut string.[3] Combining Equations 1 and 2 gives:

$$f = \frac{1}{2\sqrt{m}}\sqrt{\frac{T}{L}} \tag{3}$$

The string tension depends on the Young's Modulus of the nyltech string. The Young's

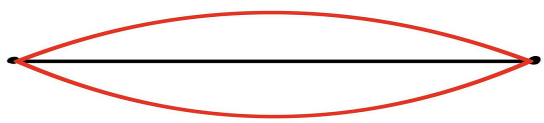

Figure 2. First harmonic standing wave on a taut string.

Modulus is given by the equation:

$$E = \frac{TL_0}{A(L_1 - L_0)} \quad (4)$$

where E is the Young's Modulus, T is the tension in the string, A is the cross-sectional area of the string, and L_0 and L_1 are the lengths of the stretched and unstretched string respectively.[2] Nylon is known to be strongly affected by both temperature and moisture content,[4] however, no studies on the effects of temperature and moisture on Nyltech have been published.

Equation 4 can be rearranged and combined with Equation 3 to give a theoretical model predicting the frequency produced by a stretched ukulele string:

$$f = \sqrt{\frac{EA(L_1 - L_0)}{4mL_1L_0}} \quad (5)$$

Since the string likely undergoes thermal expansion, the length of the unstretched string as a function of temperature will also affect the tension in the stretched string. For simple materials, like metals, the coefficient of thermal expansion is positive and constant over a range of temperatures. For nylon with its complex structure, the coefficient of thermal expansion is, unusually, negative and varies with temperature.[4] The effect of temperature on nyltech is unknown, making it difficult to integrate the coefficient of thermal expansion into the model. Instead, the length of the unstretched string with be measured over the range of temperatures, and the change in length at various temperatures will be accounted for directly, giving a final equation for the theoretical frequency of a ukulele string as a function of the Young's Modulus and unstretched string lengths at a given temperature:

$$f = \sqrt{\frac{E_{(T)}A(L_1 - L_{0(T)})}{4mL_1L_{0(T)}}} \quad (6)$$

This model assumes that the thermal expansion of the ukulele body is negligible compared to the thermal expansion of the string, so the stretched length of the string (L_1) on the ukulele is considered constant for the range of temperatures at which a ukulele is typically played.[1]

A nyltech ukulele string was used to test the theoretical model (Equation 6) of how the temperature of a taut string affects the frequency at which it vibrates when plucked. The thermal expansion and Young's Modulus of the string will be measured over a range of temperatures, and the predicted frequency compared to the measured frequency of the ukulele at those temperatures.

II. METHODS

To determine the thermal expansion and the Young's modulus of the ukulele string at different temperatures, two lengths of note G ukulele string were cut and hung with weights of 1.000 kg and 20 g, respectively. The two strings with hanging weights were put into a temperature-controlled container along with a thermometer and reference meter, as shown in Figure 3. The strings were marked at the top and bottom (circled in red) so that their lengths at each temperature could be easily

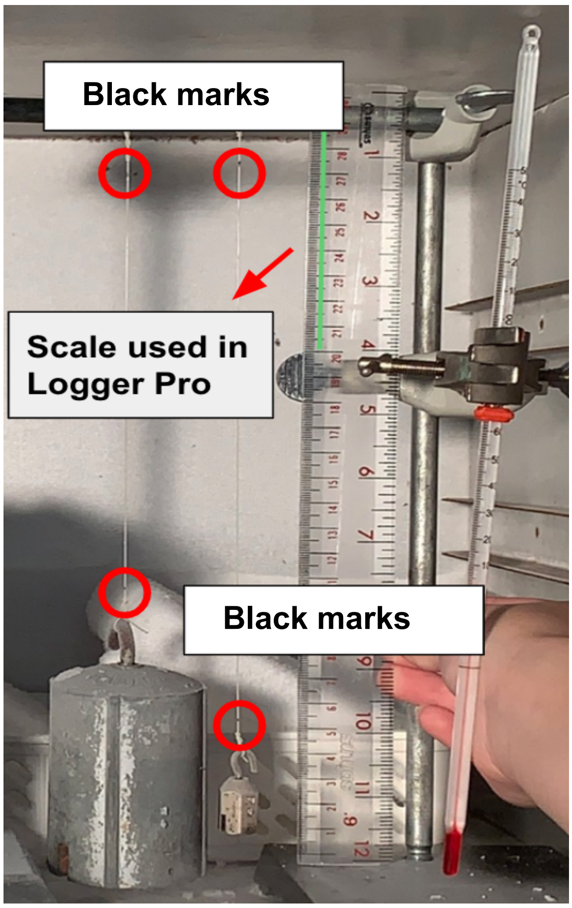

Figure 3. The stretched (left, with 1 kg) and unstretched (right, with 20 g) strings in the temperature-controlled container.

measured. The ukulele was also placed in the container.

After the apparatus was set up, the temperature was set to 60°C and left for 24 hours as the nyltech string settles over time into a stressed state, and also to reduce the moisture level to a minimum, as the moisture level of nyltech is expected to affect its Young's Modulus, as nylon is one of its components.[4] The moisture content of the string was not measured, but it was assumed to be near zero and stay constant during the data collection process.

After 24 hours passed, the temperature was set to 67°C. Twenty minutes after the temperature reached 67°C, the container door was opened, three photos of the strings were taken for later analysis, and the ukulele G string was plucked and the sound recorded with a Vernier microphone set to 0.5 seconds and 100,000 samples per second. This was done as quickly as possible to reduce temperature changes and the entry of humid air. This process was repeated for temperatures down to -10°C.

The pictures of the strings at each temperature were analyzed using Logger Pro to determine the lengths of each string at each temperature. The 20g weight kept the string straight but with negligible tension and is effectively unstretched, so any changes in its length can be considered due only to thermal expansion. For the string hung with the 1 kg weight, changes in length are due to the temperature causing changes to both the Young's Modulus and to thermal expansion, but the effect of thermal expansion can be factored out using the length of the unstretched (20 g) string at that temperature. A Fast Fourier Transform (FFT) was performed on the recorded sound of the ukulele at each temperature, as shown in Figure 4, and the frequency of the peak amplitude determined.

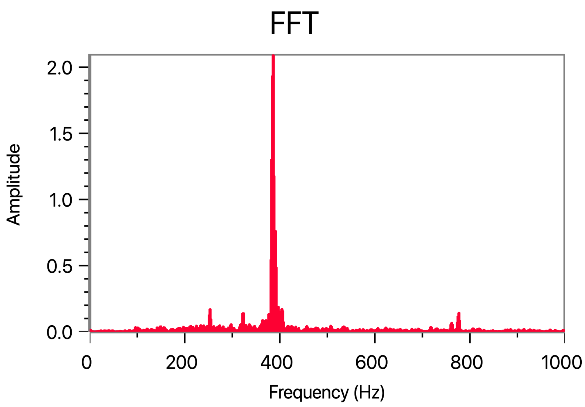

Figure 4. Sample FFT from 40.5°C.

The length of the stretched string on the ukulele (L_l) was measured. The diameter of the string was measured with a micrometer under no tension at room temperature. A Poisson's ratio[3] of 0.4, was used to calculate its cross-sectional area at the tested temperatures. It was assumed that Poisson's ratio for Nyltech approximates nylon, and since this is a second order effect, a small difference would have little impact on the results. Finally, the mass of a known length of string was measured with an electronic mass balance.

III. RESULTS AND DISCUSSION

Figure 5 below shows the length of the string with 20g changing with temperature, while Figure 6 shows the length of the string with 1kg changing with temperature. Both show a positive linear correlation between temperature and string length, as expected.

The increase in length of the unstretched string (20 g) shows the thermal expansion of the string. The fact that the length of the stretched string increased at a relatively greater rate than the unstretched string indicates that both thermal expansion and strain due to tension contributed to the lengthening of the stretched string. It should be noted that the data for -10°C appears to not follow the trend. The reason for this is not clear.

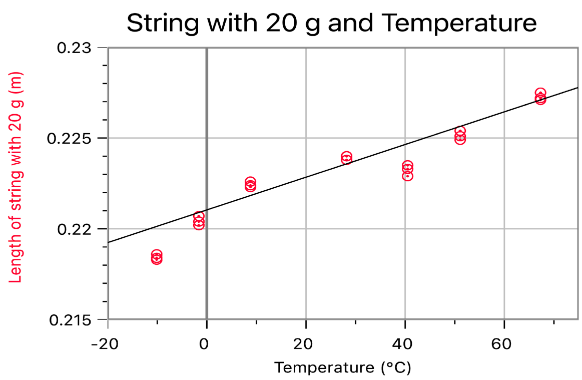

Figure 5. Temperature and the length of the string with the 20 g hanging weight.

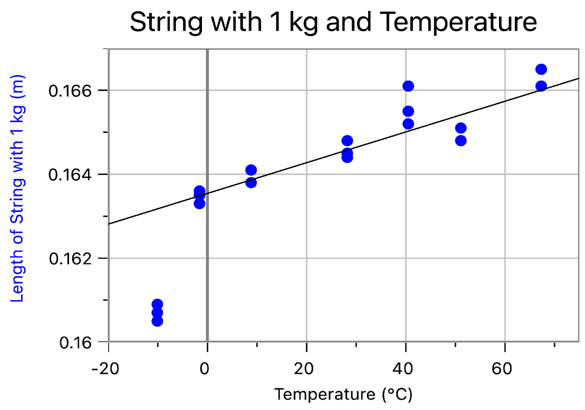

Figure 6. Temperature and the length of string with the 1kg hanging weight.

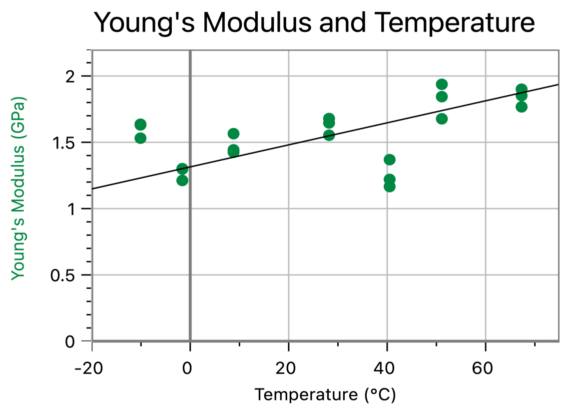

Figure 7. Graph of temperature and Young's Modulus of nyltech ukulele string.

From the data in Figures 5 and 6, the Young's Modulus was calculated and is shown in Figure 7. The Young's Modulus shows a slight increase with increasing temperature. The Young's Modulus of nylon decreases with increasing temperature in this range.[4] The two unknown materials forming the Nyltech composite seem to have the effect of keeping the Young's Modulus more constant over this range.

Figure 8 shows the frequency predicted from the theoretical model (Equation 6) and the measured frequency of the ukulele at those temperatures. The measured frequency of the ukulele decreases slightly with increasing temperature, as expected. The calculated values of frequency, while clearly showing higher variability, agree with the measured values quite closely. Unsurprisingly, the calculated frequencies for -10°C and 40.5°C, differ more significantly from the measured frequency.

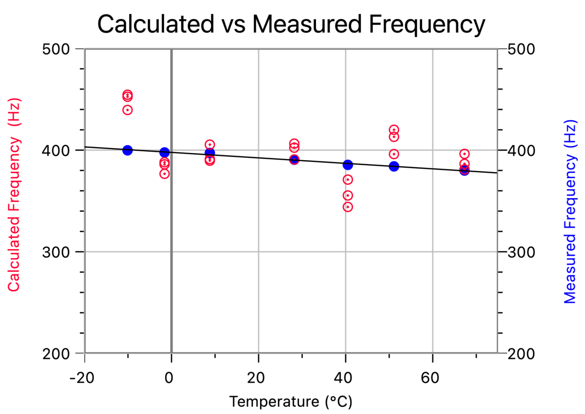

Figure 8. Graph of temperature and the calculated and measured frequency values of the ukulele string.

These results indicate that the theoretical model derived in Equation 6 is valid and that the effect of temperature on the frequency of a stretched nyltech string can be predicted given data on its thermal expansion and Young's Modulus as a function of temperature.

The use of relatively short string lengths due to the limited size of the temperature-controlled container led to difficulty in making accurate length measurements, and hence high variability in the results. However, the results support the theoretical model and suggest that this method for measuring thermal expansion and Young's Modulus is valid for nyltech stringed instruments over the temperature range of -10°C to 67°C.

IV. CONCLUSION

The frequencies of a plucked nyltech ukulele string decreased with increasing temperature. The length of the string increased due to thermal expansion, while Young's Modulus was found to increase slightly as temperature increased. The theoretical model developed, based on thermal expansion and Young's Modulus, was shown to accurately predict the measured frequencies. This indicates that the method of measuring the lengths of a heavily weighted string and a lightly weighted string at different temperatures is shown to be a valid method for measuring the thermal expansion and Young's Modulus of the nyltech string.

V. REFERENCES

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