Deformation and Energy Loss in a Basketball Bounce

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Abstract

The bounce of a basketball against the ground was recorded using a high-speed camera. The effect of the total deformation of the wall of the ball on the energy loss during impact was studied for impact speeds ranging from 3.2 ms^{-1} to 14 ms^{-1}. It was found that the energy loss increased non-linearly with increasing total wall deformation.

Keywords: basketball, bounce, compression, deformation, energy loss

I. INTRODUCTION

In basketball, dribbling or passing involves bouncing the ball on the ground. When a ball is dropped or thrown onto the ground, it will compress, as shown in Figure 1. An intuitive feel for energy loss when bouncing a basketball is part of playing well, and is developed over long hours of practice. We find that percentage energy loss is not independent of the impact velocity.

A basketball is a spherical wall filled with air. Typically, the wall of a basketball is made of multiple layers of material, including rubber, leather, nylon.[1] The composition of a Wilson basketball is shown in Figure 2. The basketball used here is made by a different manufacturer, but is expected to be similar.

Figure 1. An example of the compression of a bouncing ball.

During the bounce, the ball compresses to the shape of a truncated sphere as it makes contact with the ground. A loss of energy occurs due to hysteresis as the walls deform and then regain their original shape. As the ball leaves the surface after the impact, it returns to its original spherical shape with a lower magnitude of velocity than before the initial contact.

Previous work analyzed the coefficient of restitution in squash balls bouncing off a wall. Limpijankit found that a higher impact velocity corresponded to a lower coefficient of restitution, *COR*, of the squash ball, with a negative linear relationship.[2] Roux and Dickerson also found that the initial velocity and coefficient of restitution

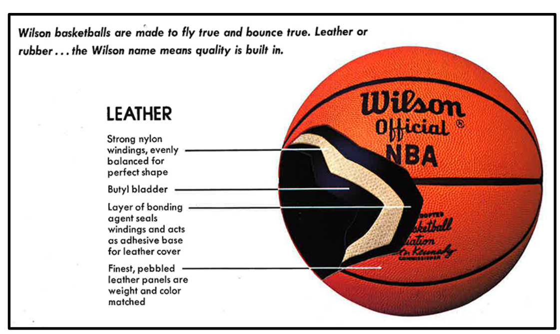

Figure 2. The material composition of a Wilson basketball.[1]

exhibited a negative nonlinear relationship when analyzing a tennis ball bouncing from a wall.[3] This suggests that a greater initial velocity would lead to more energy lost during the bounce. A possible explanation is that in collisions with greater impact velocity, there is more deformation of the wall of the ball, and therefore more energy loss due to hysteresis.

A number of researchers have found relationships between impact velocity and COR. However, none were found to have determined how the total amount of bending in the wall of a ball during a bounce relates to the loss of energy during the bounce. A model is developed here for the relationship between the total amount of bending of the wall during a bounce and the energy loss of a basketball.

Energy loss can be modeled as the difference in kinetic energy prior to and after the bounce. Kinetic energy is defined as,

$$E_k = \frac{1}{2}mv^2 \tag{1}$$

Determining the total amount of bending in the wall of the ball is more complex. Figure 3 shows a ball during its compression phase, with H the compressed diameter, R the radius, r the radius of the compressed part of the ball on the surface, and x the distance from the center of the ball to the surface. Constructing a tangent at the edge where the compressed surface meets the ground gives us the angle of bending of the wall of the ball, θ, which can be expressed as,

$$\theta = \sin^{-1}\left(\frac{r}{R}\right) \tag{2}$$

gives the angle of bending at the edge of the footprint of the ball on the floor during its bounce.

The radius of the footprint, r, shown in Figure 4 is,

$$r = \sqrt{R^2 - (H-R)^2} \tag{3}$$

Therefore, the effective amount of bending experienced by the walls of the ball, D, can be represented by the integral,

$$D = \int_0^r 2\pi r\, dr \cdot \theta \tag{4}$$

where the integral of $2\pi r\, dr$ is the total amount of bending of the wall from radius 0 to a circumference with radius r during a bounce, and θ is the angle at which each circumference is bending. Substitution of θ gives us the final integral

$$D = \int_0^r 2\pi r \cdot \sin^{-1}\left(\frac{r}{R}\right) dr \tag{5}$$

As a result, we are able to quantify the total amount of bending, D, of the walls of a ball during its compression when bounced.

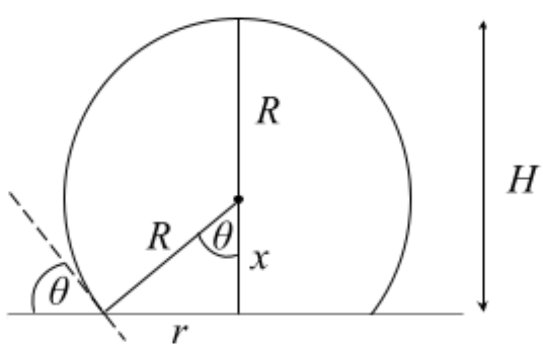

Figure 3. Compression of a bouncing ball during its bounce.

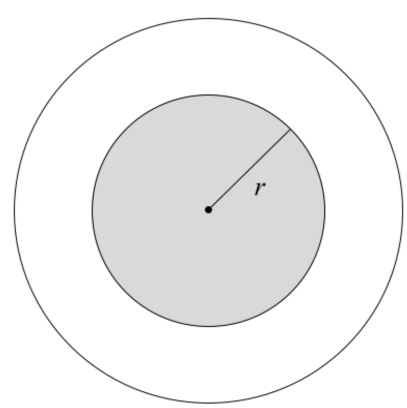

Figure 4. Bottom view of a ball during its maximum compression.

II. METHODS

The ball used was a *Tarmak* size 7 basketball with a mass of 0.609 kg, and air pressure of 7.5 psi, as per NBA regulations. A 960 fps camera was set up to capture the motion of the ball during the bounce. The camera was positioned to capture the entire motion of the ball's bounce, including at least 5 frames before and after contact with the ground. The camera was also placed as close to the ground as possible to reduce any vertical distortions. The experimental setup is shown in Figure 5.

For each trial, the basketball was dropped or thrown with negligible rotation, to prevent the "grip-slip" effect, which may influence the energy loss of the ball during its bounce.[4] The ball was launched by hand, so that it could simulate the behavior of a basketball in play, and impacted the ground with velocities ranging from 3.2 ms^{-1} to 14 ms^{-1}. This was repeated for 25 trials.

Video analysis was used to measure the velocities of the ball prior to and after bouncing, and the compressed height *H,* as shown in Figure 6. The scale, shown by the green line, was set as 0.2403 m, equivalent to the diameter of the ball. The top of the ball was tracked each frame from 5 frames before it impacted. Once the ball reached its maximum compression during its contact, the height of the compressed ball H was measured from the image. The position of the ball was recorded again for five frames after leaving the floor. This resulted in the position-time graph shown in Figure 7. The initial and final velocities of the ball were found by generating a line of best fit for the points before and after impact.

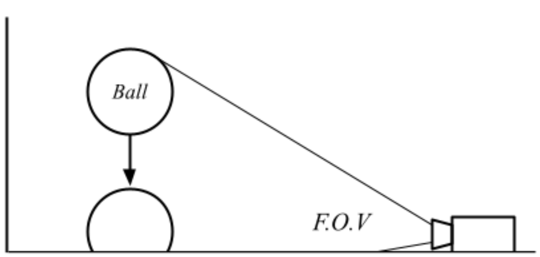

Figure 5. The experimental setup.

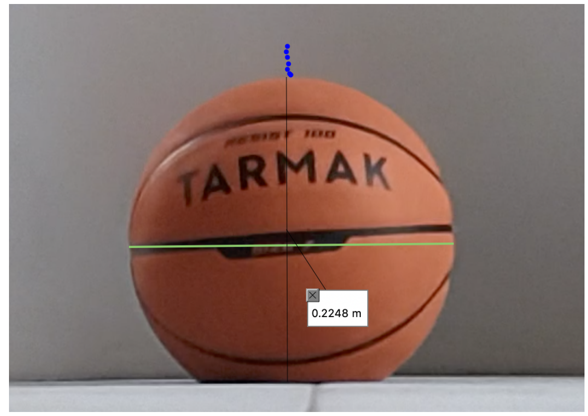

Figure 6. Sample video analysis of basketball during bounce.

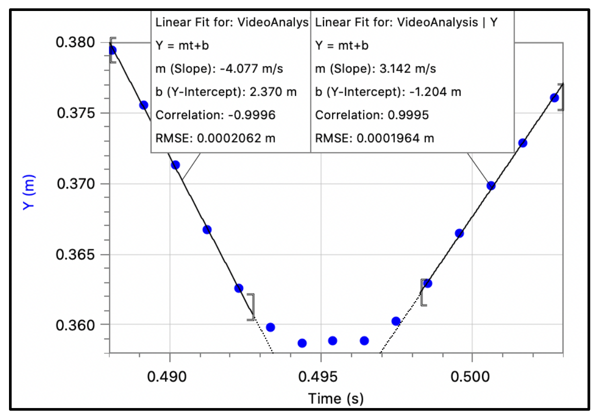

Figure 7. Sample graph showing the vertical position component of the ball against time.

III. RESULTS AND DISCUSSION

It can be seen in Figure 8 and Equation 6 that the results of this experiment show a negative linear relationship between impact velocity V_i and the coefficient of restitution,

$$COR = -(0.019 \pm 0.0028)\, V_i + (0.90 \pm 0.024) \quad (6)$$

For the tested range, the COR of a basketball bounce decreases by -0.019 for every 1 ms^{-1} increase in impact velocity. This supports the prediction that a basketball that impacts a surface with higher velocity, and therefore compression, will lose more energy during the collision.

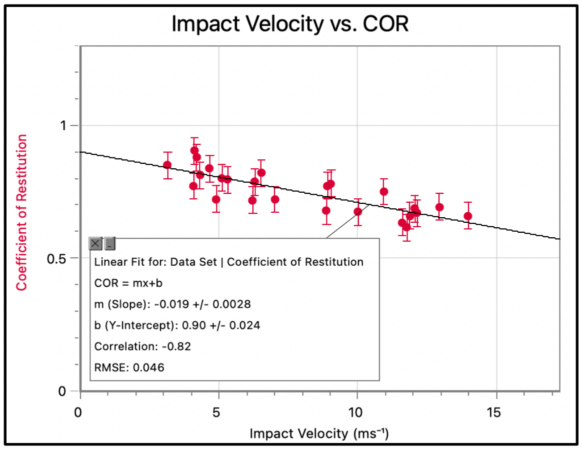

Figure 8. A negative linear relationship between impact velocity and coefficient of restitution is shown.

Despite Limpijankit and Roux using a squash ball[2] and tennis ball[3] respectively, the results can be compared as the behaviors of these balls during a bounce are similar. The squash ball exhibited a negative linear correlation between COR and impact velocity, supporting the validity of the results for the basketball here.[2] The tennis ball results were presented as a non-linear relationship, but also showed that as the impact velocity increases, the COR decreases.[3]

Following this trend, the relationship between total bending of the wall in a basketball, from Equation 5, and energy loss is shown in Figure 9. As the amount of deformation increases, the energy loss increases with a strongly non-linear trend,

$$\Delta E_k = (840 \pm 20\ kJ\ m^{-2} rad^{-1}) D^{(2.38 \pm 0.01)} \quad (7)$$

It should be noted that Equation 7 is an empirical relationship, observed for the range tested here. It is unknown if Equation 7 is valid for impact velocities outside of the tested range. Further research at a greater range of velocities is suggested.

The trend shown in Figure 9 agrees with existing theory. It has been observed through the behavior of a squash ball, tennis ball, and a basketball, that as impact velocity increases the coefficient of restitution decreases. A higher impact velocity

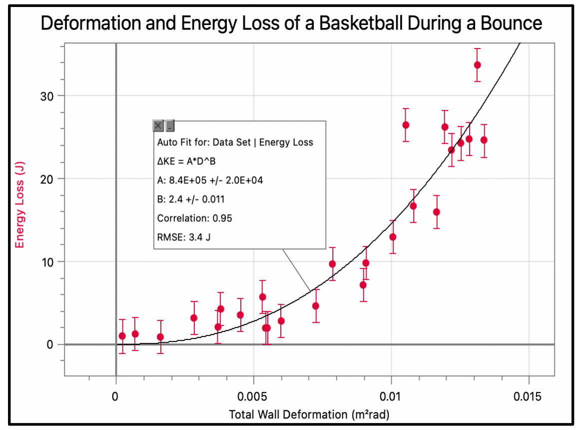

Figure 9. The total wall deformation in a ball during a bounce against the loss of energy, showing a positive nonlinear relationship.

contributes to higher total deformation, which in urn suggests higher energy loss. However, it was predicted that this relationship would be linear, assuming that the bending in the wall of the ball would increase the energy loss proportionally.

These results suggest energy loss increases in a non-linear manner; energy loss increases at an increasing rate as the angle of the wall deformation during the impact increases. Further research is needed to directly measure the energy loss due to hysteresis during bending of a basketball wall over the range of angles experienced during the bounce of a basketball at typical playing speeds.

Further investigation is also suggested into how the rotation of the ball during a bounce affects energy loss. In basketball, passes are often made by bouncing the ball at an angle to the ground. It is important to understand how this will affect the loss of linear kinetic energy during the bounce.

IV. CONCLUSION

For the range of impact velocities in basketball bounces tested, the total deformation of the wall of the ball follows a positive nonlinear relationship with the energy loss during the bounce. This suggests that increasing wall deformation angle leads to an increasingly greater loss of energy.

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