

Temperature, Frequency, and Young's Modulus of a Ching

Nicholas Jankovic

*International School Bangkok,
39/7 Samakee Road, Pak Kret,
Nonthaburi, Thailand, 11120
Email: nick@jankovic.com*

Abstract

The frequency and Young's modulus of a Thai ching were studied at temperatures ranging from -20°C to 47°C. It was found that both of these values decreased with temperature. This study also demonstrates the validity of a method of determining the Young's modulus of a material using a percussion instrument.

Keywords: Ching, cymbal, bell, temperature, frequency, Young's modulus

I. INTRODUCTION

The ching (ฉิ่ง) is a Thai hand-held percussion instrument that is used in many traditional performances. It consists of two small brass, bell-shaped cymbals connected by a thin rope and is played by striking the two together. For the ching used in this study, shown in Figure 1, each cymbal has a diameter of 5.3 cm and mass of 220g.

It is well established that an instrument's temperature affects its pitch; there's a difference between playing in an airconditioned concert hall or outside on a hot day. Every instrument responds differently to an increase in temperature. Wind instruments, for example, often increase in pitch, while string instruments typically decrease in pitch.¹ A band that tunes its instruments and then moves to a new environment may find them to be out of tune with each other again. This study aims to investigate the effect of temperature on the frequency of a ching.



Figure 1. The ching used for this research

When a ching is struck, it vibrates at a certain frequency—known as the resonant frequency—which corresponds to the pitch of the sound produced. The resonant frequency (f) of a bell is dependent on several factors, including thickness (t), diameter (d), Young's modulus (E), density (s), and Poisson's ratio (m). These can be combined into Equation 1, where k is a constant determined by the characteristics of the bell:²

$$f = \frac{kt}{d} \sqrt{\frac{E}{s(1-m^2)}} \quad (1)$$

Although temperature doesn't appear in Equation 1, all factors in this equation except k are dependent on it to some degree. However, the effect of temperature on Poisson's ratio is negligible for the range of temperatures tested,³ and the coefficient of linear expansion of brass is very small, so the changes in thickness, diameter, and density are negligible as well.⁴ This leaves only Young's modulus, a measure of a material's elasticity, as being significantly temperature-dependent for the range tested here.

Temperature and Young's modulus have a negative relationship for most metals.⁵ For example, Young's modulus decreased linearly with increasing temperature for naval brass (Figure 2), an alloy that is likely similar to the unknown brass type of a ching. Thus, a reduction in Young's modulus caused by rising temperature is expected to result in a decrease in the frequency of the ching.

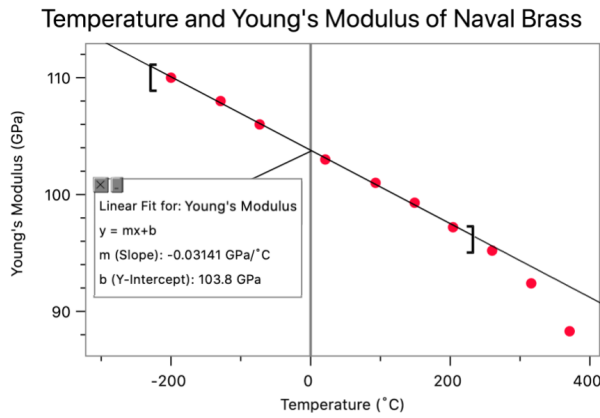


Figure 2. Temperature and Young's modulus of naval brass,⁵ with a linear fit applied to the data between -200 °C and 200 °C.

Since the precise composition of the ching is not known, its Young's modulus is also not known. Moreover, as k isn't known for the ching, Young's modulus cannot be determined from Equation 1. However, the relative change in Young's modulus caused by changes in temperature can be determined.

Equation 1 can be re-written as,

$$f = h\sqrt{E}, \text{ where } h = \frac{kt}{d\sqrt{s(1-m^2)}} \quad (2)$$

It follows that relative Young's modulus, E_R , at different temperatures can be expressed as,

$$E_R = \frac{E_2}{E_1} = \frac{\left(\frac{f_2}{h_2}\right)^2}{\left(\frac{f_1}{h_1}\right)^2} = \frac{(f_2 h_1)^2}{(f_1 h_2)^2} \quad (3)$$

Since the effect of temperature on h is negligible for the range tested, $h_1 = h_2$. Therefore,

$$E_R = \left(\frac{f_2}{f_1}\right)^2 \quad (4)$$

Changes in frequency can be used to determine the relative change in Young's modulus as a function of temperature.

While bells are known for their clear notes, they will often oscillate at two or more frequencies simultaneously, due to axial asymmetries caused during the manufacturing process.⁶ These frequencies interfere and, when similar enough, are perceived by the human ear as a "beat," an alternating increase and decrease in the amplitude of the sound pressure wave.⁷

In this paper, the effect of temperature on the frequency of the ching will be studied, and then used to determine the relative change in Young's modulus of the metal. The beat frequencies of the ching will also be analyzed.

II. METHODS

A ching was brought to various temperatures ranging from -20 °C to 47 °C, then struck while holding the rope so that both cymbals emitted a clear, long-lasting sound. The sound was recorded at 100,000 samples per second. A fast fourier transform (FFT) was performed on roughly 2 seconds of data in each trial, and the frequency of each of the four resonance modes was graphed with temperature.

This data was then used to find the relative Young's modulus (E_R) using Equation 4, with the frequency at -20 °C as f_1 and all other frequencies as f_2 . Finally, the line of best fit was found for the naval brass data (Figure 2) and used to calculate the Young's modulus at the specific temperatures used in this study. These values were relativized in the same fashion as above for comparison with the ching data.

III. RESULTS AND DISCUSSION

Ringling the two hand-crafted, asymmetric ching cymbals at the same time gave some interesting results. The sound pressure-time graph (Figure 3) shows the sum of three frequencies close to 4500 Hz varying in amplitude with regular beats of frequencies 6 Hz and 42 Hz.

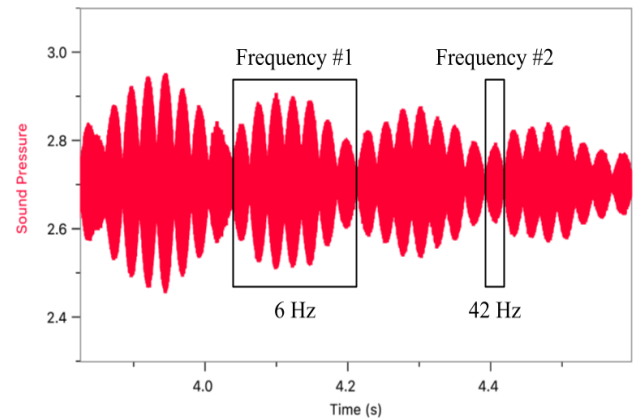


Figure 3. Sample sound pressure-time graph from striking the ching at a temperature of -18°C.

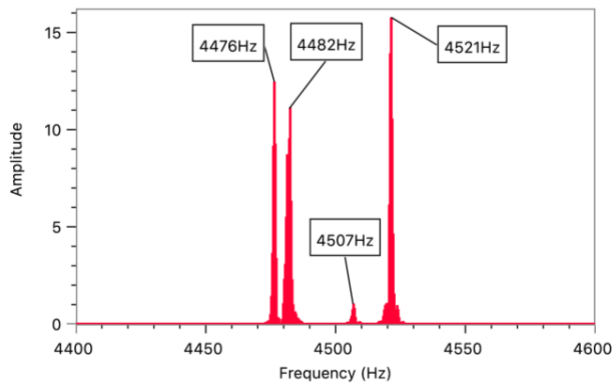


Figure 4. Sample FFT from -18°C . The two lower frequency peaks were emitted by one cymbal, and the higher two were emitted by the other.

Figure 4 shows the FFT of the sound of the ching shown in Figure 3. The sound waves of the first cymbal, with frequencies of 4476 Hz and 4482 Hz, interfere to create a clearly perceived beat of 6 Hz, equal to the difference between the frequencies. This corresponds to Frequency 1 in Figure 3.

A beat could not be heard for the second cymbal due to the small amplitude of its lower frequency of 4570 Hz. Instead, the higher frequency of the second cymbal, 4521 Hz, interferes with the average frequency of the first cymbal, bringing about the 42 Hz frequency seen in Figure 3. This was not perceived as a beat, but as adding to the dissonance of the sound heard. Although the frequencies and amplitudes changed with temperature and the location on the rim where the ching was struck, a pattern similar to that in Figures 3 and 4 was typically observed.

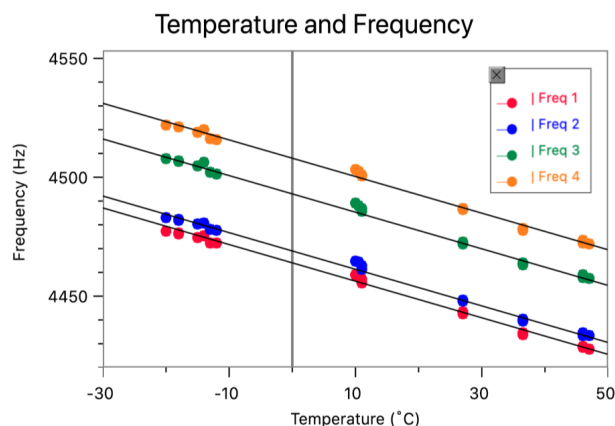


Figure 5. Temperature as a function of frequency. All slopes are $-0.77 \text{ Hz}^{\circ}\text{C}^{-1}$. Frequencies 1 and 2 were emitted by one cymbal, while 3 and 4 were emitted by the other.

Figure 5 shows the four frequencies of the ching as they vary with temperature. Each mode of resonance had the same relationship with temperature. The frequencies of these modes decreased as temperature increased, with a slope of $-0.77 \pm 0.06 \text{ Hz}^{\circ}\text{C}^{-1}$.

The relative Young's modulus of the ching was also found for each data point, as shown in Figure 6. The change in relative Young's modulus due to temperature, shown in blue, is $-0.00034 \text{ }^{\circ}\text{C}^{-1}$, while the relative change for naval brass, in red, is $-0.00030 \text{ }^{\circ}\text{C}^{-1}$. For context, the Young's modulus of naval brass at 27°C is 103.0 GPa, as shown previously in Figure 2.

This method of finding a material's Young's modulus as a function of temperature requires none of the usual equipment or complex procedure. In order to determine the values of Young's modulus at various temperatures, it's enough to know the value at one specific temperature and have a bell that's made from the same material. Then the Young's modulus at any other temperature can be determined by measuring the frequency of the bell at that temperature.

This method can likely be used to determine the Young's modulus at various temperatures for other percussion instruments, such as tuning forks, xylophones, and cymbals, as long as the following two conditions are met. First, the mathematical relationship between Young's modulus and frequency of that instrument must be known. Second, Young's modulus at a certain temperature must be known. Further research is suggested confirming whether this method may be applied to other materials and instruments.

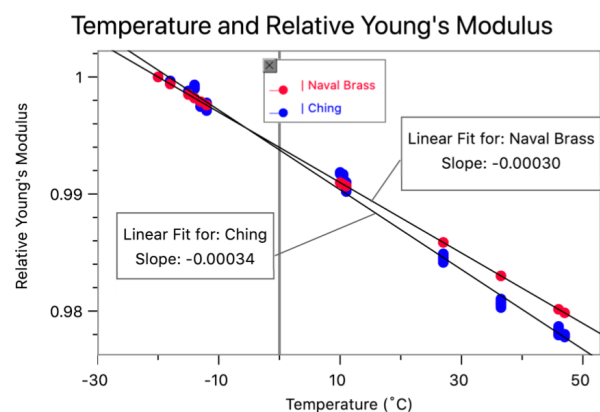


Figure 6. Temperature and relative Young's modulus of the ching and naval brass.

IV. CONCLUSION

The frequencies produced by a Thai ching were all found to decrease with increasing temperature, with a slope of $-0.77 \pm 0.06 \text{ Hz}^\circ\text{C}^{-1}$. The relative Young's modulus of the ching's brass was also found to decrease with temperature at a rate of $-0.000340^\circ\text{C}^{-1}$. This value closely matches the known rate of change for naval brass. Finally, this study outlined how relative Young's modulus values found using the frequency of an instrument can be converted to actual values if a single initial value is known. This provides a simple way of finding Young's modulus values for a range of temperatures.

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