Impact Velocity and Coefficient of Restitution of a Table Tennis Ball

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Abstract

The impact of a table tennis ball upon a paddle was recorded using a high-speed camera. The effect of the ball's initial impact velocity on its coefficient of restitution was determined for impact velocities ranging from 1.4 ms^{-1} to 20.2 ms^{-1}. It was found that the coefficient of restitution decreased linearly with increasing impact velocity.

Keywords: table tennis ball, coefficient of restitution, impact velocity

I. INTRODUCTION

When a table tennis ball impacts a stationary paddle, the ball can be clearly seen to leave the paddle with a slower velocity. The ratio of the final velocity and the incoming impact velocity is described as the coefficient of restitution (COR). In this study, the COR of a table tennis ball is tested using a typical range of in-game velocities.

The collision of a ball with a stationary object will always cause the ball to bounce with a lower velocity after every collision. This decrease after each bounce can be explained by the conversion of kinetic energy into sound and heat during the impact. The COR is defined as the ratio of the final velocity to the initial velocity, modeled by the equation,

$$COR = \frac{v}{u} \quad (1)$$

where v and u represent the final velocity and initial velocity respectively.[1]

The coefficient of restitution represents the elasticity of a collision. A collision that has no kinetic energy lost is fully elastic, resulting in a coefficient of 1. A collision that results in a loss in all kinetic energy is known as an inelastic collision, represented by a coefficient of 0.[1]

Previous work by Kawazoe & Suzuki,[2] who also investigated the effect of impact velocity and the COR of a table tennis ball against a rubber paddle, showed a negative linear trend as shown in Figure 1.

The graph illustrates that as the impact velocity increases, the COR will decrease linearly. Kawazoe & Suzuki's study still falls short in some aspects: velocities below 8 ms^{-1} were not tested, nor any velocities between 8 ms^{-1} and 21 ms^{-1}. The surface of impact is also not clearly identified. Due to the limitations of Kawazoe & Suzuki's work, this study will further add to our knowledge of table tennis ball behavior over the full range of in-game velocities.

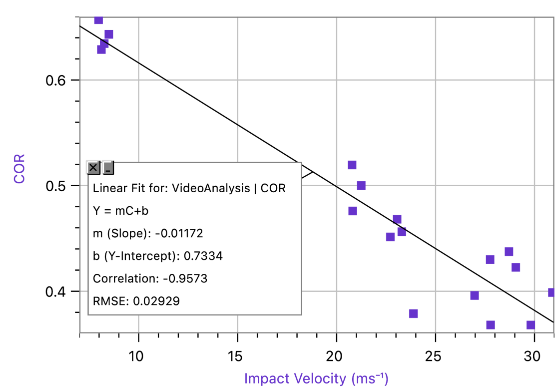

Figure 1. Kawazoe & Suzuki's results of the effect of the impact velocity of a table tennis ball on its COR.[2]

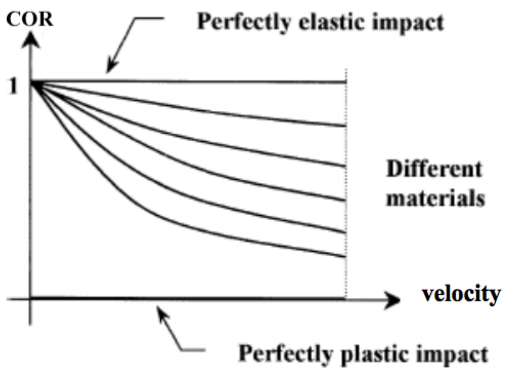

Figure 2. Goldsmith's proposed COR as a function of impact velocity.[3]

A linear trend, as shown in Figure 1, may not be an accurate representation of the ball's COR when approaching very low velocities. As impact velocity approaches 0 ms^{-1}, the deformation of the ball approaches 0. As a result, there is no energy loss, producing a COR approaching 1.[3] This theory is explored in Gilardi's paper. Previous work by Goldsmith from Gilardi's paper, who investigated impacting spherical bodies, concluded that COR will not decrease linearly as impact velocity increases.[3] The general model of this relationship for different material balls can be seen in Figure 2.

Goldsmith's trend can be modeled by an inverse power function with a y-intercept of 1, shown in Equation 2:

$$COR = 1 - Au^{B} \tag{2}$$

where u is the initial impact velocity.[4] Based on this equation, as the impact velocity increases the COR is hypothesized to decrease less significantly.

While Goldsmith's model mainly addresses theoretical considerations, Kawazoe & Suzuki's is based on their empirical data. The intent of this study is to investigate which of the two models, a linear trend or inverse power trend, is a better representation of the COR for typical in-game velocities.

II. METHODS

A RITC 729 C-3 table tennis paddle was secured flat onto a hard surface with the backhand side of the paddle faced up, as shown in Figure 3. A contact area of 12.5 cm by 8.1 cm was marked in the center of the paddle. A Tibhar SynTT 40+3 ball of mass 2.70 ± 0.01 grams was launched at different velocities onto the contact area.

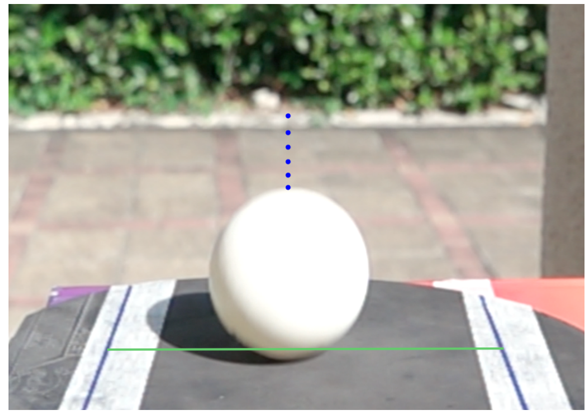

Figure 3. Sample frame-by-frame video analysis, depicting the analyzed frames as the ball contacts the paddle during its descent.

A Sony RX10 camera recorded the motion of the ball's impact on the paddle at 1000 frames per second. LoggerPro's video analysis was used to calculate the COR of 40 different impact velocities.

Five frames each before and after the frame of impact were used to determine the impact and exit velocity of the ball, as shown in Figure 4. Only impacts of the ball that had an impact angle of 90 ± 5° with the paddle and were within the area of contact were analyzed.

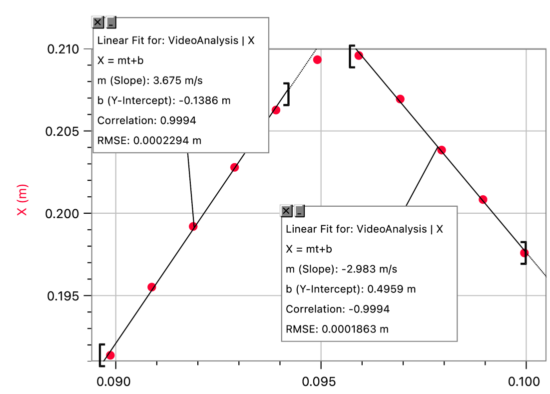

Figure 4. Position-time graph resulting from the ball's displacement in Figure 3.

III. RESULTS AND DISCUSSION

Firstly, the linear model is examined. Shown in Figure 5, the COR of the ball decreases linearly by 0.018 for every 1 ms^{-1} increase in the impact velocity.

This relationship is best represented by:

$$COR = -0.019\,u + 0.87 \qquad (3)$$

Equation 3's slope of -0.018 is similar to Kawazoe & Suzuki's slope of -0.012, adding to the confidence of the results. The difference in Kawazoe & Suzuki's slope can be explained by difference in their ball and surface.

However, the y-intercept of 0.858 is not in accordance with theory, since energy loss would be expected to approach 0 as the ball's impact velocity approaches 0 ms^{-1}. Hence, a curve fit proposed by Goldsmith may better account for the expected COR at very low impact velocities, shown in Figure 6 and Equation 4:

$$COR = 1 - 0.095\,u^{0.55} \qquad (4)$$

Although it is observed that a curve is not a suitable fit for impact velocities under 4 ms^{-1}, the negative trend is in accordance with Goldsmith's predicted model. From Equation 2, an impact velocity of 0 ms^{-1} is expected to have a COR value of 1, which is satisfied with Equation 4.

The curve from Figure 6 indicates that the trend of deformation of the ball may differ significantly at impact velocities of less than 1.4 ms^{-1}. Therefore, further research is suggested investigating the COR of the ball at impact velocities below 1.4 ms^{-1}. This further research may supplement the data needed to confirm whether an inverse power relationship is a suitable model for the COR of a table tennis ball.

Regardless, a linear model still provides a better correlation between this study's data and the fit when

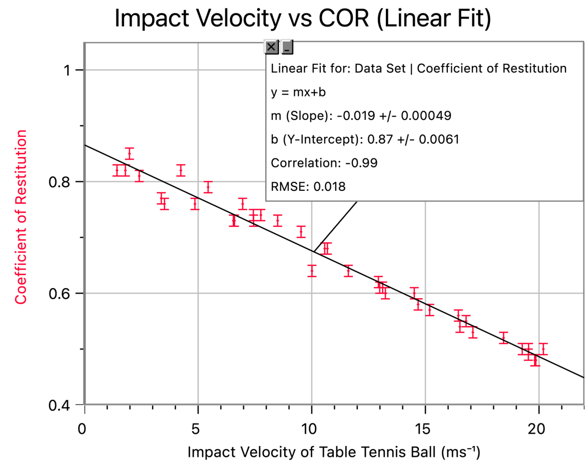

Figure 5. The negative linear trend between the initial velocity of the table tennis ball and the COR.

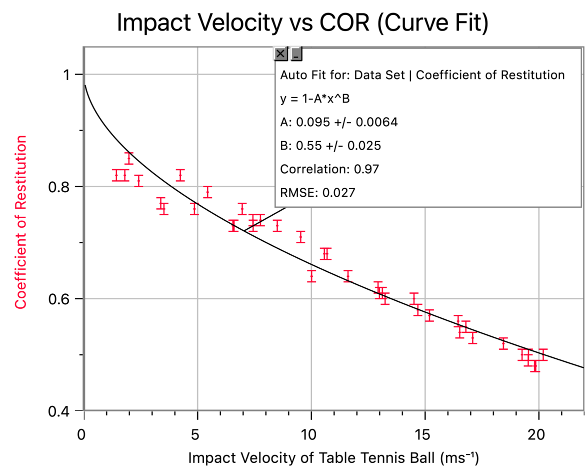

Figure 6. The curve trend between the initial velocity of the table tennis ball and the COR of the collision.

compared to the curve. It should be noted that the linear model cannot be extrapolated for impact velocities below 1.4 ms^{-1} with any confidence, hence the need for further research.

Another suggested area of further research would be to investigate the COR of the forehand side of the paddle. The forehand side is made of a different type of rubber, which propels the ball faster than the backhand side[5]. Developing an apparatus to measure ball deformation during its impact would also help to develop a full model of the energy loss of the ball during impact.

IV. CONCLUSION

A negative linear relationship is shown to best model the relationship between the impact velocity of a table tennis ball and its COR for a velocity range between 1.4 to 20.2 ms^{-1}. For an increase of 1 ms^{-1} in impact velocity, COR will decrease by 0.019. It is possible that the change in COR will increase significantly for impact velocities under 1.4 ms^{-1}.

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