The Hot Chocolate Effect in Water and in Foam

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Abstract

Frank S. Crawford's application (1982) of the resonance in air model to find the speed of sound in liquids and foams in glass and ceramic cylinders is tested for water and albumen foam at different depths in a large beer mug.[1] It is found, in contradiction of popular belief, that the model fails at most depths for water in a beer mug, but holds for the foam at all depths. No end correction is required and the velocity of sound in the albumen foam is that expected for a 90% air-filling fraction with water, and is density dependent.

***Keywords*:** hot chocolate effect, speed of sound, foam, density

I. INTRODUCTION

For an oscillating column of air in a cylinder that is open on one end, the velocity of sound in air may be calculated from the primary resonant frequency using the equation,

$$v = 4fL \tag{1}$$

where v is the speed of sound, f is the primary resonant frequency in hertz (s^{-1}), and L is the length of the column of air in the cylinder.[2] This equation has been derived from the equation $v = f\lambda$, where λ is the wavelength of sound produced. Since the length of the waveform produced in the air column is a quarter of the full wavelength of sound produced, λ can be replaced with $4L$ to give equation 1. Similarly, the speed of sound in air may also be calculated from the second or third resonant frequencies. In general, for the nth harmonic frequency, the speed of sound in air may be calculated using equation 2 below,

$$v = \frac{4}{2n-1} fL \tag{2}$$

When air oscillates in a cylindrical container, an end correction of typically one third of the cylinder's diameter is observed because some air beyond the open end of the cylinder takes part in the oscillations.[3]

For an oscillating column of water, Frank S. Crawford argued in a 1982 paper,[1] by analogy with a vibrating air column, that the length of the column of liquid should be one-quarter wavelength for the lowest mode of sound waves in the liquid, both for the case of a bubble-free liquid, when the liquid is uniformly filled with bubbles (before they have had time to float to the top) or when there is more air than liquid as in a foam. He calculated the speed of sound in water, with and without bubbles, on the basis of this assumption, but apparently did not verify his results for all depths of water.

Preliminary investigations have shown that while Crawford's assumption does give the speed of sound in water at one particular depth, it does not hold true for all depths.

Crawford does not acknowledge in his paper the possibility of coupling between the glass walls and the water. The density of water and the density of glass are comparable (in the ratio 1: 2.5), which may result in resonant coupling between the glass walls and the water. Many other papers in the literature and accounts on the web make no mention of this. Crawford further claimed that there would be no end correction observed in the oscillating system.[1]

If Crawford's end correction claim holds for oscillations in a foam, and there is no coupling between the foam and the walls of the container, then the speed of sound in albumen foam will be given by equation 1 at all depths.

II. METHODS

Testing the Resonance Model for Water

Initial trials were conducted using a thin-walled cylindrical glass filled with water. Tapping the bottom of this glass (wall thickness 2.25 mm) yielded FFT graphs showing no clear discernible peaks.

A 22.5 oz (640 ml) beer mug was then firmly secured over a gap of 3 cm between two solid surfaces. The mug filled with different depths of water was tapped with a sandstone pestle on the bottom to produce a sound which was recorded with a Vernier microphone using LoggerPro at 100,000 samples per second. The mug was tapped 8-12 times with the pestle per trial. The sound recordings were analysed using an FFT algorithm to find the peak resonant harmonic frequencies for each trial. Figure 2 shows an example of a typical FFT graph obtained.

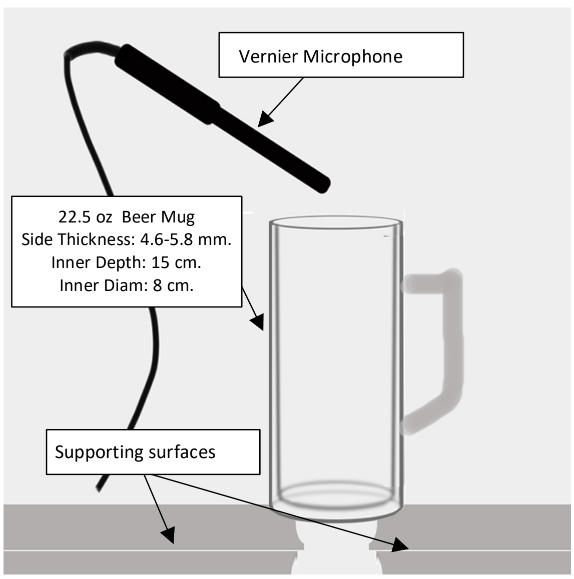

Figure 1. The experimental setup with the beer mug secured between two surfaces to allow the bottom to be tapped.

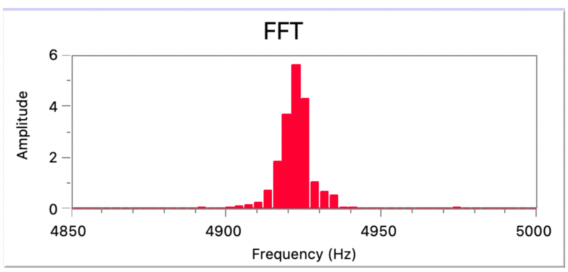

Figure 2. A sample FFT for water depth 0.075 ±0.003 m, showing a peak frequency of 4922 Hz for the first harmonic.

Testing the Resonance Model for Albumen Foam

Another set of trials was conducted with albumen foam. Egg-white (albumen) was beaten using an electric mixer until it formed a foam. The albumen foam was poured into the mug at different depths and the bottom of the mug was tapped with a pestle as before to find the apparent speed of sound in albumen foam using the first and second harmonic frequencies. Sample FFTs are shown in figure 3.

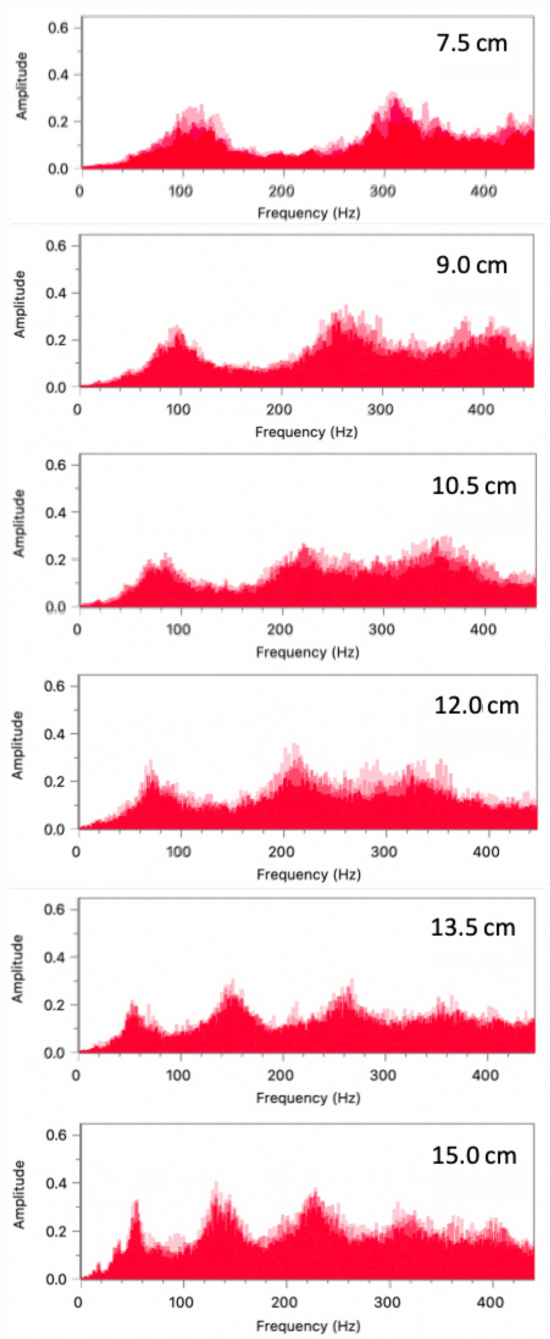

Figure 3. FFTs obtained for the mug containing albumen foam at different depths.

Density and Speed of Sound in Albumen Foam
Finally, albumen foam of different densities was made by beating the albumen with an electric beater for different lengths of time and filling the mug to the rim, 15 cm depth. The speed of sound was calculated and compared with the literature. FFTs for this set of trials is shown in figure 4. Although the peaks were narrow and well defined for the trials conducted with water, the peaks were much broader for the albumen foam, possibly due to a lack of consistency in the uniformity of the mixture.

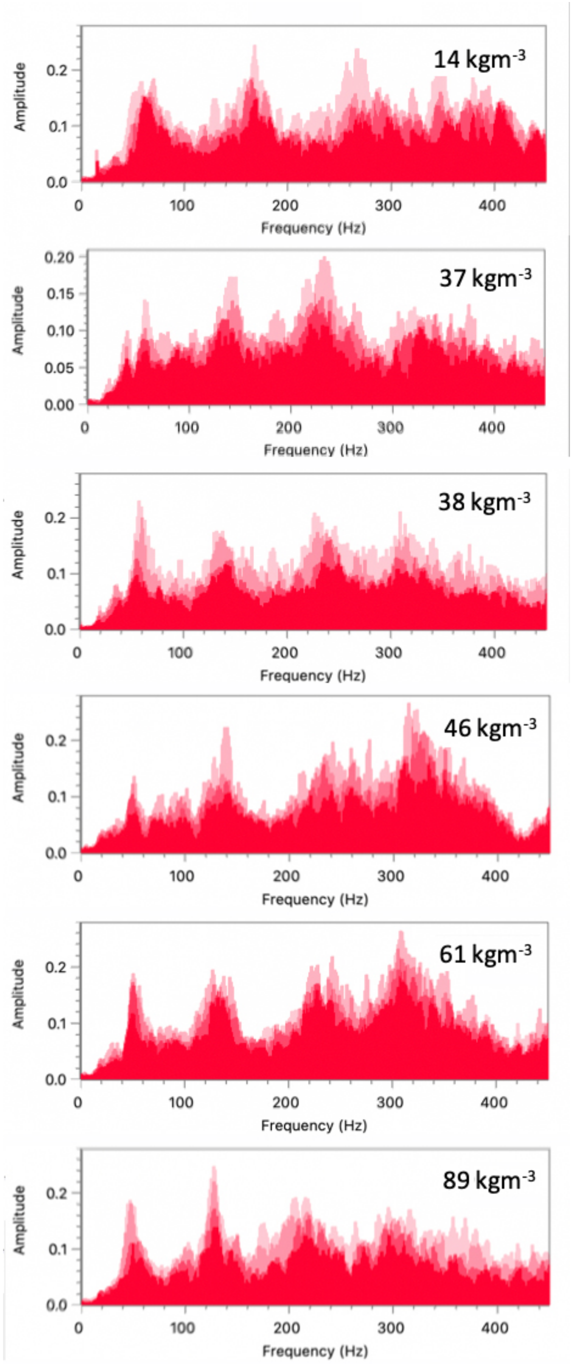

Figure 4. FFTs obtained for the mug filled with albumen foam of different densities.

III. RESULTS & DISCUSSION

Testing the Resonance Model for Water
The frequencies of the first harmonic at different water depths were used to calculate the apparent speed of sound in water, using the formula $v = 4fL$. The graph plotting the apparent speed of sound against the depth of water in the beer mug is shown in figure 5. The relationship obtained is clearly inconsistent with Crawford's findings.

The actual speed of sound in water (1500 ms^{-1} at room temperature) is returned (by interpolating using the best fit curve) only when the depth is 0.082 m, or 55% of the total depth of the beer mug. It is clear from figure 5 that Crawford's method for calculating the speed of sound in water does not return the correct value for the speed of sound when a glass mug containing water is nearly full.

Crawford's model could possibly be used to compare, for instance, the velocity of sound in brine and water if a reference depth is established, where the speed of sound in water is accurately returned in a large thick-walled glass mug. However, Crawford's resonance model fails for water in a large beer mug because significant coupling occurs between the glass walls and the water. The correct value for the speed of sound in water appears when the mug used here is a little over half full. Many articles on the web misrepresent the resonance[2], saying that the correct value for the speed of sound in water is returned when the mug is nearly full, which in this case is incorrect.

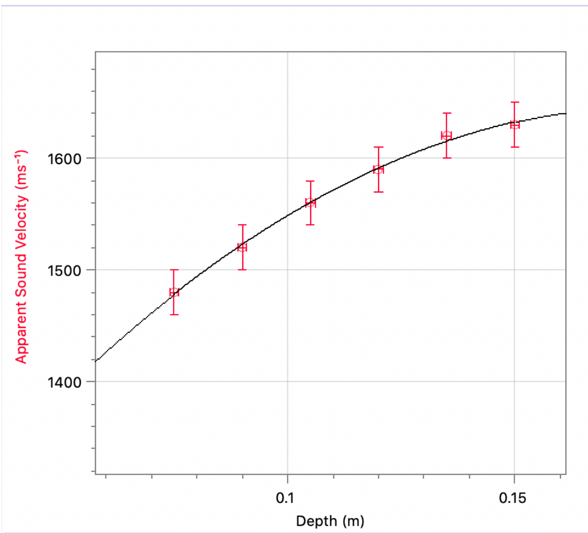

Figure 5. The apparent speed of sound in water varies with the depth of water in the 15 cm deep beer mug.

Testing the Resonance Model for Albumen Foam

The same experiment was carried out again, but this time with albumen foam of density 200 ± 20 kg m^{-3}. As shown on figure 6, the values returned for the speed of sound in albumen are constant within errors when using the first and second harmonic frequencies respectively. Apparent velocities calculated using the second harmonic are systematically slightly lower than the velocity values calculated using the first harmonic. The small second order effect occurs in many similar instances.[4]

As can be seen, wall coupling is not a significant factor for resonance in albumen foam, as the density of glass is much greater than the density of the albumen foam used in this experiment.

The data was examined using an arbitrary end correction value to check whether having an end correction improved the consistency of the apparent calculated velocities using the first five harmonic frequencies. It was found that inserting an end correction to each harmonic made the velocity values less consistent, therefore we can assume that there is no end correction within this system.

The Density/Speed of Sound relationship in Albumen Foam

Finally, the density of the albumen foam was varied and the frequencies when the full mug was tapped were obtained from gaussian analysis of the FFT graphs produced. These FFTs had peaks which were less clear when compared to the FFT graphs for the experiment with albumen foam at different depths. It is unclear why this was the case.

The relationship between the density of the albumen foam and the speed of sound in the foam is shown in figure 7. It is clear that the speed of sound in the foam decreases with increasing density.

For foams, the density of the foam is often characterized by the fraction of the foam that is comprised of air. Using the data points given in figure 7, the air fraction, f_a, in the albumen foam was derived based on the equation:

$$\rho = \rho_a \cdot f_a + \rho_{albumen} \cdot (1 - f_a) \qquad (3)$$

where ρ is the density of the albumen foam mixture, ρ_a is the density of air, and $\rho_{albumen}$ is the density of albumen. The density of albumen was measured as 1017 ± 2 kgm^{-3}. Rearranging the variables in equation 3, we obtain the following expression in terms of f_a:

$$f_a = \frac{\rho - \rho_{albumen}}{\rho_a - \rho_{albumen}} \; . \qquad (4)$$

Equation 4 was used to calculate the air fraction f_a for all values of the density of the albumen foam shown in figure 7.

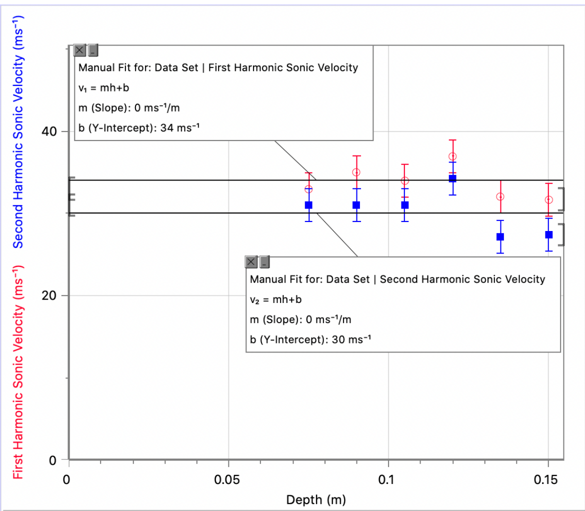

Figure 6. The speed of sound in albumen foam, calculated from the first and second harmonics, stays constant for all depths in the 15 cm mug.

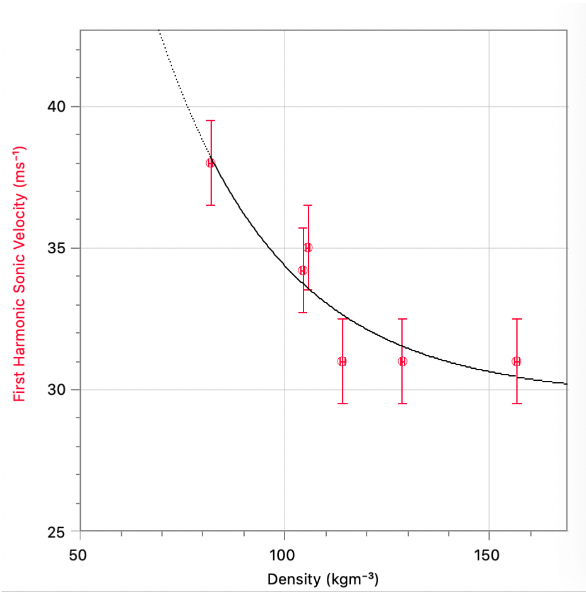

Figure 7. The sonic velocities in foam of varying density, calculated from the first and second harmonic frequencies.

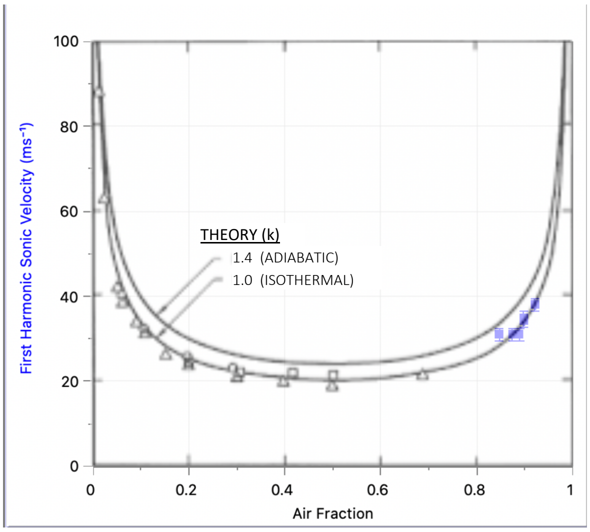

Figure 8. The data points obtained from this investigation (on the right in blue) overlaid against the relationship given by Brennen.[4]

The data is plotted in figure 8, overlaid on a graph obtained from Christopher E. Brennen's paper on Cavitation Bubble Dynamics (1995).[4] As figure 8 clearly shows, the data covers a region of the graph which did not have any experimental data points supporting the proposed relationship. Plotting the data points obtained in this investigation on the model using by Brennen[4] corroborates the model used in the literature for determining the relationship between air fraction and speed of sound in a foamy mixture.

Further research in this field is suggested to determine the effect of the coupling of glass walls with the fluid or foam in both thick- and thin-walled glass cylinders. Research to experimentally confirm Brennen's results over the full range of foam densities is needed, as well as using foam of a more uniform density.

IV. CONCLUSION

It has been shown that Crawford's model of resonance for water with bubbles in it (the Hot-Chocolate Effect) in a thick-walled glass mug is not valid, due to coupling between the water and the walls. However, Crawford's model is valid for the resonance of foam in a mug for which there is no appreciable coupling between the oscillations of the foam and the wall. Crawford's assertion that there is no end correction for the physical system, has been confirmed for albumen foam in a glass mug. For foam, the literature model for sound velocity as a function of air fraction does hold as the data points from this investigation match the isothermal curve predicted by Brennen.[4]

V. REFERENCES

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