# Coefficient of Restitution of a Squash Ball

# Marvin Limpijankit

International School Bangkok, 39/7 Samakee Rd, Pakkret, Nonthaburi, 11120, Thailand Email: marvin.limpijankit@gmail.com

#### **Abstract**

The coefficient of restitution of a Double Yellow Dot squash ball was examined at impact velocities ranging from 13 to 29 m/s. A high-speed camera was used with video analysis to record and analyze the motion of a squash ball as it collided against a squash court wall. It was found that there is a negative linear relationship between impact velocity and COR within this velocity range.

Keywords: squash ball, impact velocity, coefficient of restitution

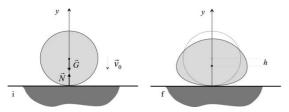
### I. INTRODUCTION

The coefficient of restitution (COR) of a squash ball influences the properties of its motion during collisions. COR refers to the ratio between the velocities of two colliding masses before and after the collision has occurred. If no energy is lost in the collision, then the collision is said to be perfectly elastic (COR of 1). If energy is lost, then the collision is an inelastic collision (COR value of <1, depending on how much energy is lost). For a collision in which a ball collides with a wall, the equation for the COR is, <sup>1</sup>

$$COR = \frac{v}{u} \tag{1}$$

where v is the velocity of the ball after the collision and u is the velocity of the ball immediately before the collision.

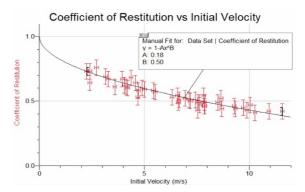
As the ball collides against the wall, the ball is deformed as the center of mass continues to travel in the direction of the hit while the wall exerts an opposing force, N, on the ball<sup>2</sup>, as illustrated in Figure 1. Some energy is lost due to internal friction during the deformation, while the rest of the energy is transformed from kinetic energy to elastic energy as the ball is compressed, and then transforms back into kinetic energy as it bounces off the wall.



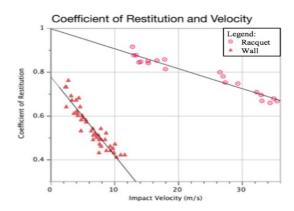
**Figure 1.** Deformation of a ball colliding with a surface.<sup>2</sup>

An increase in impact velocity increases deformation. Thus, more energy is expected to be lost to hysteresis resulting in a decrease in COR.

Previously, studies have found that an increase in impact velocity results in a decrease in the COR of tennis ball collisions with various surfaces. One such study, by Roux and Dickerson, found a negative relationship between the impact velocity, ranging from 2 to 12 m/s, and the COR of a tennis ball being hit against a wall,<sup>3</sup> as shown in Figure 2.



**Figure 2.** COR and Initial Velocity graph for a tennis ball colliding against a wall.



**Figure 3.** COR and Impact Velocity for a tennis ball colliding with a tennis racquet and a wall.

Another study, by Sukpraprut, showed a negative linear relationship between the impact velocity and COR of a tennis ball colliding with a racquet.<sup>4</sup> A comparison of his results to the results of the first study are shown in Figure 3.

This investigation focuses on the relationship between the initial impact velocity and the COR of a Dunlop Double Yellow Dot squash ball being hit against a squash court wall. Velocities were tested within the range at which squash is typically played at the amateur level, approximately 10 to 30 m/s. Following Roux's and Sukpraprut's results, it is expected that as the impact velocity is increased, the COR of the squash ball will decrease.

### II. METHODS

A 600 frames-per-second video camera was set up facing parallel to a squash court wall as shown in figure 4. The temperature of the Dunlop Double Yellow Dot ball used in the investigation was controlled at  $25.0 \pm 0.5$ °C, measured using an infrared gun, by submerging the ball in a water bath.

At an approximate distance of 2 meters from the wall, the squash ball was hit at varying game-realistic speeds (10 – 29 m/s) at an angle perpendicular (within 6°) to the plane of the wall. Multiple trials were conducted until the squash ball's temperature increased above 25.5°C, then the ball was submerged in the water bath to cool. Through video analysis, shown in figure 5, a position-time graph was constructed as shown in figure 6.

Initial and final velocities were calculated using the slope of the 10 x-position points immediately before and after the collision, respectively. Trials in which the ball impacted at an angle greater than 6° from perpendicular were discarded. In total, 33 collisions were analyzed.

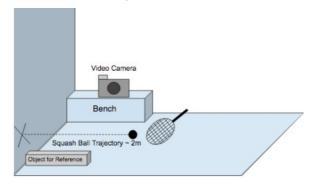
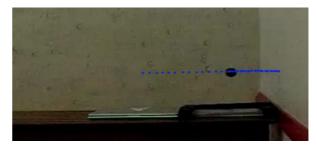
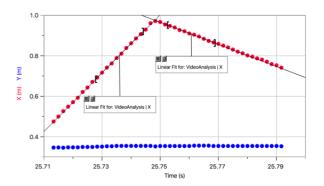


Figure 4. Experimental Set-Up.



**Figure 5.** Sample Video Analysis showing the position of the ball in each frame.



**Figure 6.** Position-time graph to calculate u and v.

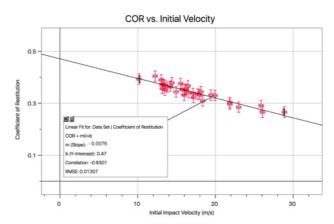
## III. RESULTS AND DISCUSSION

Equation 2 illustrates the relationship between initial velocity and COR for the squash ball colliding against the wall,

$$COR = 0.47 - (0.0076) u \tag{2}$$

where u is the initial velocity of the ball in meters per second.

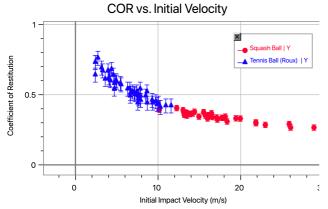
As evident in figure 7, there is a negative linear relationship between the initial impact velocity and COR of the squash ball for the tested range of impact velocities. Within the range tested, the COR of the squash ball decreases by 0.0076 for



**Figure 7.** COR has a negative linear relationship with initial velocity in the range 10 - 29 m/s.

every 1 m/s increase in speed. This validates the prediction that as the squash ball collides against the wall with greater velocity, more energy is lost during the collision.

It is unlikely that the relationship in equation 2 would be applicable to shots much beyond the tested range since this would imply that an impact velocity approaching 60 m/s would yield a COR approaching 0 and thus not bounce off the wall.



**Figure 8.** Results from this experiment graphed with those of Roux and Dickerson<sup>3</sup>.

When graphically compared to the results of Roux's experiment, it is evident that the two sets of data cross over at initial speeds of 10 - 12 m/s. As seen in figure 8, within this specific range, the points on each graph intersect one another at COR values of approximately 0.4, as the graph transitions from lower impact velocities to higher ones. While it is important to note that these data sets describe two different balls, it may suggest that had this experiment been conducted with lower impact velocities, a curve trend may have been apparent, with the COR approaching 1 as initial impact velocity approached 0.

This curve would account for the fact that the magnitude of the slopes of both data sets are different, with that of the tennis ball being much larger. Additionally, a curve similar to the one illustrated by the graph would be theoretically consistent, as less impact velocity should result in less deformation and thus less energy loss due to friction and a larger COR (approaching a perfectly elastic system as velocity decreases more and more).

This paper only looks at one factor that affects the bounce of a squash ball. Other factors that relate to the COR of a squash ball have not yet been investigated. Examining the effect of the collision angle or the magnitude of ball spin on the COR

would be interesting. Another important factor which has not been fully described is the effect of ball temperature on COR. Finally, to extend the findings of this paper, a wider range of initial impact velocities could be investigated. Investigating lower and higher impact velocities may prove helpful in validating the proposed curve trend discussed in the earlier sections of this paper.

#### IV. CONCLUSION

A negative linear relationship between the initial impact velocity and COR was found for a squash ball impacting a wall with speeds ranging from 13 to 29 m/s. The COR of the squash ball was found to decrease linearly with increasing impact velocity.

### **REFERENCES**

- 1. Madden, D., et al. (2007). *Physics: A contextual approach*. Port Melbourne: Heinemann.
- 2. Güémez, J. and Fiolhais, M. (2013). From Mechanics to Thermodynamics-analysis of selected examples. European Journal of Physics, 34(345)
- 3. Roux, A. and Dickerson, J. (2007). Coefficient of Restitution of a Tennis Ball. International School Bangkok Journal of Physics, 1(1).
- 4. Sukpraprut, Mongkol. (2015). Impact Velocity and Coefficient of Restitution for a Tennis Ball Impacting a Tennis Racquet. International Scholastic Journal of Science, 9(1).