

Helmholtz Resonance in Korean Bells

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Abstract

Traditional Korean bells are installed above a shallow well with a gap between the bell and the ground, effectively creating a Helmholtz resonator. During installation, the size of the well and the gap is traditionally adjusted, through trial and error, until the Helmholtz frequency matches the fundamental frequency of the bell. Here, the frequency of the Helmholtz resonator in Korean bells was investigated. Models of the resonator with various dimensions were built, and their resonant frequency measured. An equation for the Helmholtz frequency of the resonator was developed, which predicts the frequency of Korean bells within 4%. The equation can be used by traditional Korean bell installers to design the well and gap to ensure that the Helmholtz frequency of the resonator matches the fundamental frequency of the bell.

Keywords: Korean bell, Helmholtz resonance, resonance well, air gate

I. INTRODUCTION

The sacred bell of King Seongdeok the Great (figure 1) is famous not only for its sound quality and long-distance delivery but also for the special acoustic device employed in its installation: the resonant well in the ground below the bell and the narrow air gap between the bell and the ground.^{1,2} Bells with these unique features are given the name Korean bell³, used to indicate traditional bells made between 700 and 1400 AD.



Figure 1 Sacred bell of King Seongdeok the Great.

The well below the bell and the narrow gap between the bell and the ground play a similar role to the sound box in stringed instruments. The Korean bell's "sound box" consists of an air cavity and an air gate as illustrated in Figure 2. The air cavity consists of 3 parts: the air inside the bell, the well, and the space between these two. The air gate is the ring-shaped gap between the bell and the ground. These combine to form a Helmholtz resonator.

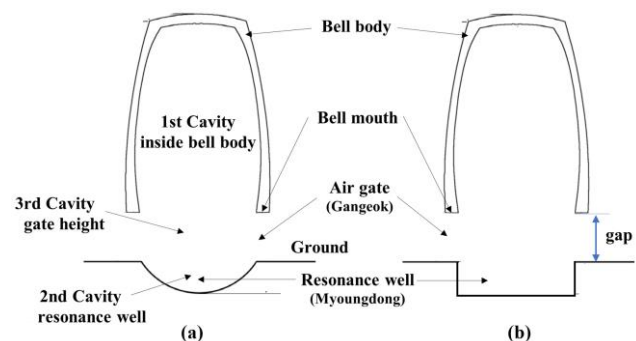


Figure 2 Helmholtz resonator of a Korean bell with (a) semi-spherical resonance well, (b) cylindrical resonance well.

The resonator of a Korean bell amplifies the sound pressure by coupling the vibration of the bell body to the surrounding air. If the frequency of the bell matches the frequency of the Helmholtz resonator, the sound of the bell will be amplified, having a profound effect on the overall sound quality of the Korean bell.⁴

Helmholtz resonance is the phenomenon of air resonance in a cavity, such as when one blows across the top of an empty bottle. The name comes from a device created in the 1850s by Hermann von Helmholtz, the Helmholtz resonator (figure 3a).⁵

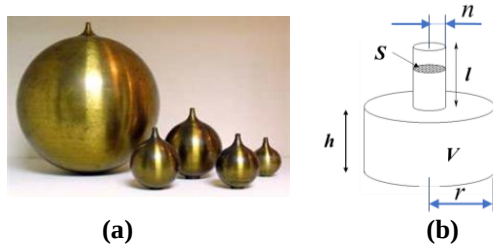


Figure 3 Helmholtz resonator, (a) a brass, spherical Helmholtz resonator based on his original design, circa 1890-1900, (b) standard model of a Helmholtz resonator.

The resonant frequency (f) of a standard Helmholtz resonator (figure 3b) is:⁵

$$f = \frac{c}{2\pi} \sqrt{\frac{S}{V l_{eq}}} \quad (1)$$

where c is the speed of sound in air, S is the surface area of the hole, V is the volume of air in the resonator's body, and $l_{eq} (= l + 1.45 n)$ is the effective length of the air volume oscillating through the neck.

It has traditionally been considered important to install the bell so that the frequency of the bell's Helmholtz resonator is as close as possible to the fundamental frequency of the bell itself, but determining the size of the well and air gap needed is difficult. Recent studies have determined the resonant frequency of the Helmholtz resonator of Korean bells using frequency measurement tools¹ and computer simulation⁴, but these methods are not practical for the craftsmen who typically install Korean bells.

This study aims to develop an equation for the Helmholtz frequency of a Korean bell which can be used by craftsmen to easily and accurately calculate the size of the well and air gap needed for a particular Korean bell. A series of models with various dimensions were built and the resonant frequencies determined. A modified Helmholtz resonance equation was then developed to accurately determine the measured frequencies of the models, and tested against real Korean bells.

II. METHODS

Standardized models with simplified geometries were used to model the Helmholtz resonators of Korean bells, as shown in figure 4. Only models with cylindrical wells (figure 4b) were tested. The equation for the frequency of the standard Helmholtz resonator

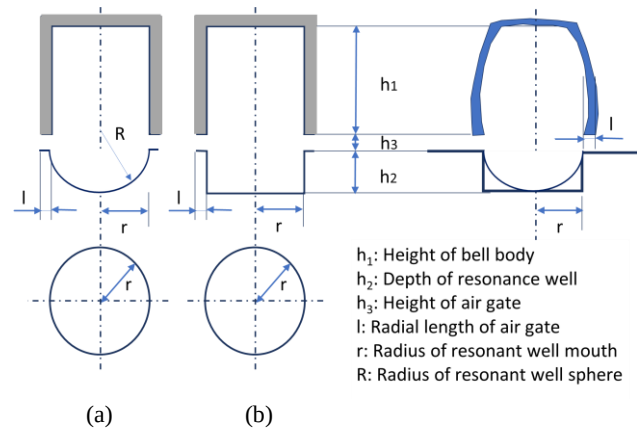


Figure 4 Standardized model of Korean bell's soundbox with (a) Semi-spherical resonance well, (b) Cylindrical resonance well.

can be modified to reflect the geometry of the Korean bell resonator. The ring-shaped air gate (fig 5a) in the bell resonator can be thought of as a truncated cone (figure 5b), since the surface area of the outer mouth is greater than the surface area of the inner mouth. Adjusting the mounting position (figure 5c) results in the standard model of the Helmholtz resonator with a truncated cone neck in place of a cylindrical neck (figure 3b).

Since the parameters defined in Eq. 1 are maintained through the three cases (figures 5a-c), the resonance frequency is expected to be the same. Therefore, the physical principle is the same and only the structural shape is different. In Figure 5, the air gate is shown outside of the volume. An outward air gate is common for Helmholtz resonators. Inward air gates also occur, as, as in the woofer hole of acoustic speakers. In this study, both types of air gate will be investigated.

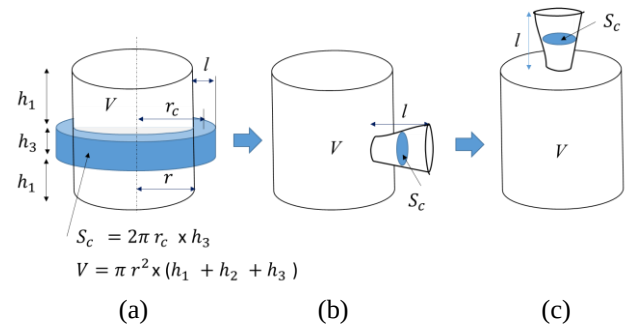


Figure 5 Transformation from Korean bell resonator to Helmholtz resonator through geometric reconstruction (a) Korean bell 'sound box' (b) Transient shape (c) Helmholtz resonator with a truncated cone; S_c is the surface area of the neutral plane in the air gate; r_c is the radius of the neutral plane, S_c .

The Helmholtz resonance frequency of a Korean bell resonator is proposed as:

$$f = \frac{c}{2\pi} \sqrt{\frac{S_c}{V l_{eq}}} \quad (2)$$

where $l_{eq} = l + \Delta r$ is the effective length of the air volume oscillating through the air gate, and Δr is the effective air diffusion length. Given the shape of the Korean bell's air gate, a new expression for the air diffusion length is proposed as:

$$\Delta r = \frac{2\alpha}{2\pi r} S_c = \frac{\alpha}{\pi r} S_c \quad (3)$$

The value of the coefficient α was determined empirically using a series of 18 models of a simplified Korean bell resonant cavity. The test samples were made with acrylic pipes with dimensions as shown in Table 1. A variety of scenarios were tested: inward vs outward air gate (A and B), various depths of the well (B1-3), and various air gate heights for different well and cavity dimensions (C-E).

Sample num.	A1	A2	A3	B1	B2	B3	C1	C2	C3
Dimension(m)									
h_1	0.30	0.30	0.30	0.30	0.30	0.30	0.10	0.10	0.10
h_2	0.00	0.10	0.20	0.00	0.10	0.20	0.00	0.00	0.00
h_3	0.03	0.03	0.03	0.03	0.03	0.03	0.01	0.02	0.03
l	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01
r	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
Air gate	Outward			Inward			Inward		

Sample num.	D1	D2	D3	E1	E2	E3	F1	F2	F3
Dimension(m)									
h_1	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.30
h_2	0.00	0.00	0.00	0.20	0.20	0.20	0.30	0.30	0.30
h_3	0.01	0.02	0.03	0.01	0.02	0.03	0.01	0.02	0.03
l	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
r	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
Air gate	Inward			Inward			Inward		

Table 1 Six sets of test samples with varying dimensions and air gaps (h_3). See figure 4 for the dimension parameters.

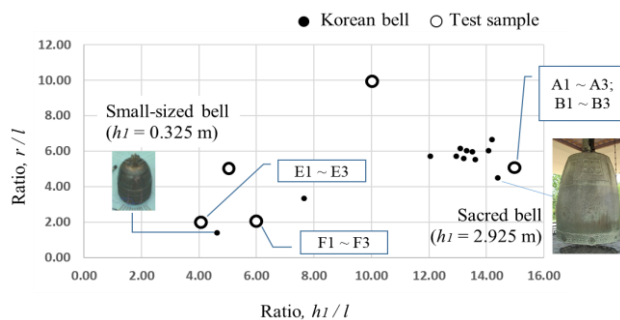


Figure 6 Comparison of geometric ratios: h_1/l vs. r/l .

When the test samples were designed, specific geometric ratios such as h_1/l and r/l (see figure 4) were considered to reflect the geometric characteristics of the range of typical Korean bells. In Figure 6, the geometric ratios: h_1/l vs. r/l for 12 Korean bells are shown, along with the test samples in Table 1. It is clear that the test samples model the geometric variance of real Korean bells, even though the sampled Korean bells' height (h_1) varied from 30 cm (Small-sized bell) to almost 3 m (Sacred bell).

A schematic diagram of the test setup for measuring the Helmholtz resonance frequency is shown in figure 7a. Figure 7b shows the actual equipment used. The sound pressure level was measured with a sound pressure sensor located inside the resonator and a reference level sound pressure for the speaker was found at the periphery of the resonator. The sound signal was generated by using a smartphone signal generator application which repeatedly generated a sweep across a frequency range from 50 Hz to 200 Hz. The measured signals were collected using NI USB-4431, and the frequency response functions were processed by using NI S/W (Sound and Vibration Toolkit) in the computer. The temperature inside the resonator was kept constant at 20°C for all tests.

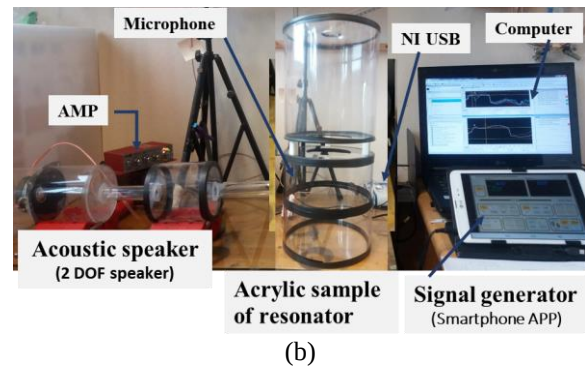
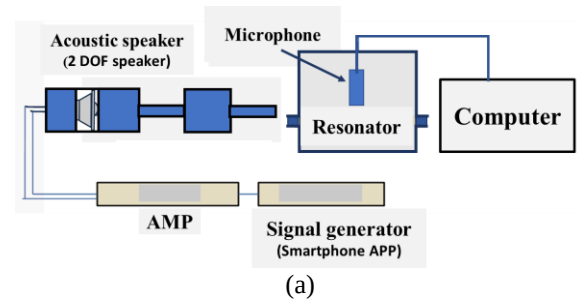


Figure 7 Measurement of the acoustic frequency response function, shown as (a) Schematic diagram of experimental setup, (b) Equipment used: the acoustic speaker and one of the test resonators with an inward air gate, $h_1 = 20$ cm, $h_2 = 20$, $h_3 = 3$ cm, $l = 5$ cm, $r = 10$ cm (Samples E1 - E3)

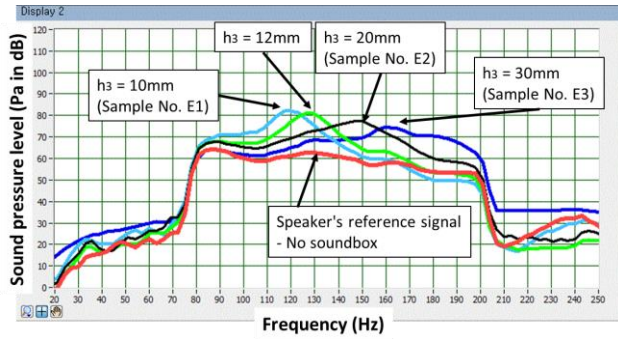


Figure 8 Frequency response function curves: Speaker reference signal (red curve); Sound pressure level inside resonator according to the gap height of air gate, h_3 ; f of E1 = 120 Hz, f of E2 = 150 Hz, f of E3 = 163 Hz.

In Figure 8, a typical set of Frequency Response Function (FRF) curves are shown. The red line shows the speaker output reference signal, measured with no resonator attached. The flatness of this curve between 80 and 200 Hz shows that the speaker produces a reference signal of constant sound pressure in the frequency range of interest. For the FRF curves with the three resonators, a peak is clearly observed, corresponding to the resonant frequency of each resonator. The FRF curves for test samples E1-3 (light blue, black, and dark blue, respectively) are shown for air gate heights (h_3) ranging from 10 mm to 30 mm, showing that as the gap height increases, the resonant frequency increases.

II. RESULTS AND DISCUSSION

The measured resonant frequencies of all the test samples were compared to the frequency predicted by equations 2 and 3 for values of α varying between 0.2 and 2.0. The average difference between the measured and predicted frequency is graphed against the different values of α tested in figure 9. The minimum error of 4.1% for the calculated values for frequency occurred for an α value of 1.3.

Figure 10 shows the measured frequencies of each of the test samples (gray) compared to the predicted frequency (blue stripes) of the sample using equations 2 and 3 for $\alpha = 1.3$, along with the percent difference between the two (green). compares the Helmholtz frequency of each sample, Test vs. Calculation by using a histogram and in addition, the error of each sample with the coefficient, $\alpha = 1.3$. The proposed equation is consistent in its predictions across all test sample dimensions, with errors varying from 1.3 to 7.2 %, with most less than 5 %.

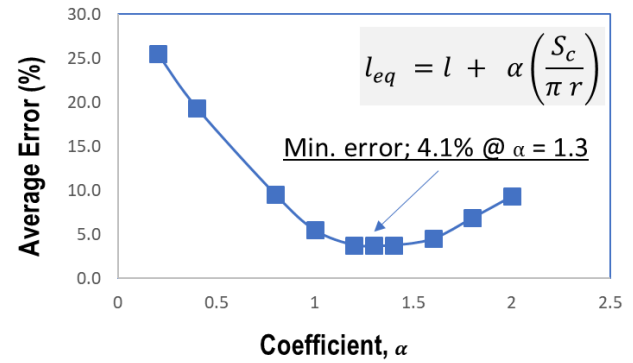


Figure 9 Average error between measured and predicted values of frequency vs. Coefficient, α . The minimum error occurs for $\alpha = 1.3$.

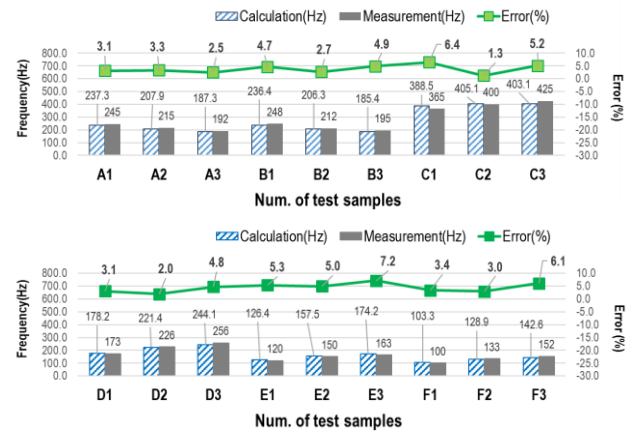


Figure 10 Measured and predicted frequencies of all test samples, with percent difference.

Geometrical parameter	Real bell	
	Sacred bell	Small-sized bell
h_1	2.9250	0.3250
h_2	0.3200	0.0000
h_3	0.5000	0.2000
l	0.2030	0.0700
r	0.9105	0.0980
Measurement	24.0	195.0
Calculation	24.8	201.3
Error ¹⁾	3.3%	3.2%

Table 2 Published dimensions and frequencies of two Korean bells^{1,6} and the frequency predicted by the proposed equation.

The proposed equation can now be used to predict the Helmholtz frequency of real Korean bells based on their measured dimensions. Table 2 shows the predicted frequencies and the published measured frequencies of two Korean bells, the Great King Seongdeok¹ and a small-sized Korean bell.⁶

The proposed equation approximates the Helmholtz frequency of two real Korean bells with errors of around 3%. This shows that the proposed equation, with its empirically determined coefficient, can accurately predict the frequency of real Korean bells. It is expected that craftsmen installing Korean bells will be able to use this equation to determine the dimensions of the well and air gap needed so that the frequency of the Helmholtz resonator matches the fundamental frequency of the bell.

IV. CONCLUSION

It has been shown that the air cavity and gap of a Korean bell acts as a Helmholtz resonator, and that the behavior can be modeled with a modified Helmholtz equation. Using a series of standardized test samples with varying dimensions, the coefficient adjusting the effective neck length of the Helmholtz resonator was found to be 1.3. Using this coefficient in the proposed equation, the frequency of real Korean bells was predicted within 3%. It is hoped that the proposed equation will be used to aid in the installation of Korean bells.

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